This question paper contains 8+2 printed pages]

Your Roll No. ....

1183

B.A. (Programme)/II **C**

MATHEMATICS—Paper II

(Geometry, Differential Equations and Algebra)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note : The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'. These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) Describe and sketch the graph of the equation : 6

$$16x^2 \quad y^2 - 32x - 6y \quad 57$$

(*b*) Find an equation of the ellipse that has its foci at (2, 1) and (2, 3) and major axis of length 6. Also sketch the graph. 6

(*c*) Consider the equation

$$x^2 + 2\sqrt{3}\ xy + 3y^2 - 2\sqrt{3}\ x - 2y = 0$$

Rotate the coordinate axes to remove the xy-term and then identify the type of the conic represented by the equation and sketch it. 6

2. (*a*) Find the equation of the sphere passing through the points (3, 0, 2), (–1, 1, 1) and (2, –5, 4) and having its center on the plane $2x + 3y + 4z = 6$.

Also find its center and radius. 6½

(*b*) (*i*) Find K so that the vector from the point A(1, −1, 3) to the point B(3, 0, 5) is orthogonal to the vector from A to the point P(K, K, K). 3

(*ii*) Determine whether the vectors

$$\vec{u} = 5\hat{i} - 2\hat{j} + \hat{k},$$

$$\vec{v} = 4\hat{i} - \hat{j} + \hat{k} \text{ and}$$

$$\vec{w} = \hat{i} - \hat{j}$$

lie in the same plane ? 3½

(*c*) (*i*) Show that the planes :

$$-2x + y + z = 0 \text{ and}$$

$$6x - 3y - 3z - 5 = 0$$

are parallel.

Find the distance between these planes. 3

(*ii*) Show that the lines

$L_1 : x = 1 + 7t,\ y = 3 + t,\ z = 5 - 3t,$

$-\infty < t < \infty$

$L_2 : x = 4 - t,\ y = 6,\ z = 7 + 2t, \quad -\infty < t < \infty$

are skew lines. 3½

3. (*a*) (*i*) Solve the differential equation : 3

$$(x^2 + y^2 + x)\, dx + xy\, dy = 0.$$

(*ii*) Solve : 3½

$$y = 3x + \log p,$$

where

$$p = \frac{dy}{dx}.$$

(*b*) (*i*) Solve the differential equation : 3

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 12y = (x-1)e^{2x}.$$

(*ii*) Solve : 3½

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x.$$

using the method of variation of parameters.

(*c*) (*i*) The number of bacteria in a yeast culture grows at a rate which is proportional to the number present. If the populaion of a colony of yeast bacteria triples in 1 hour. find the number of bacteria which will be present at the end of 5 hours. 3½

(*ii*) Solve the simultaneous equation : 3

$$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = -x.$$

$$x(0) - 0, y(0) = 0.$$

4\. (*a*) (*i*) Here

$$p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y},\ r = \frac{\partial^2 z}{\partial x^2},\ s = \frac{\partial^2 z}{\partial x \partial y},\ t = \frac{\partial^2 z}{\partial y^2}$$

Form the partial differential equation by eliminating the arbitrary constants a and b from the equation

$$2z = (ax + y)^2 + b$$

Examine whether the partial differential equation so obtained is linear or non-linear. 3

(ii) Find the general integral of the differential equation : 3

$$(z - y)\,p + (x - z)\,q = y - x.$$

(b) (i) Find a complete integral of the equation :

$$2(z + xp + yq) = yp^2.$$

using Charpit's method.

(ii) Find whether the equation : 4+2

$$x^2(y - 1)\,r - x(y^2 - 1)\,s + y(y - 1)\,t + xyp - q = 0.$$

(c) Find a complete integral of the equation : 6

$$p^2 - q^2 = x - y.$$

5. (*a*) (*i*) If

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix}, \text{ and}$$

$$\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 3 & 5 \end{pmatrix},$$

compute $\sigma^{-1}\rho\sigma$. 3½

(*ii*) Express :

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 1 & 2 & 6 & 5 \end{pmatrix}$$

as a product of transpositions. 3

(*b*) Prove that the intersection of any collection of subgroups of a group is again a subgroup. Does the result hold for union of subgroups also ? Support your answer. 6½

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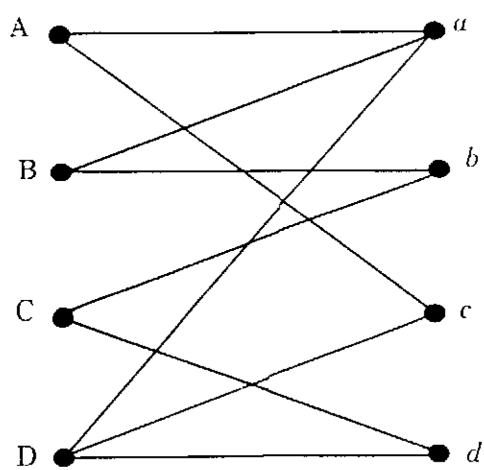
(c) Show that the set $R_1 = C[0, 1]$ of all real valued continuous functions from [0, 1] to **R** forms a commutative ring with respect to the operations + and . defined by :

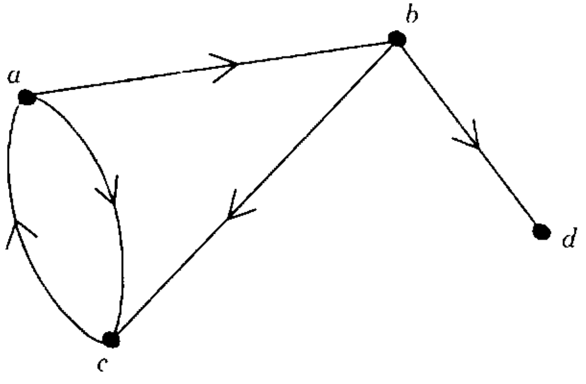
$$(f + g)(t) = f(t) + g(t) : 0 < t \leq 1$$

$$(f \,.\, g)(t) = f(t) \,.\, g(t) : 0 < t \leq 1. \qquad 6\tfrac{1}{2}$$

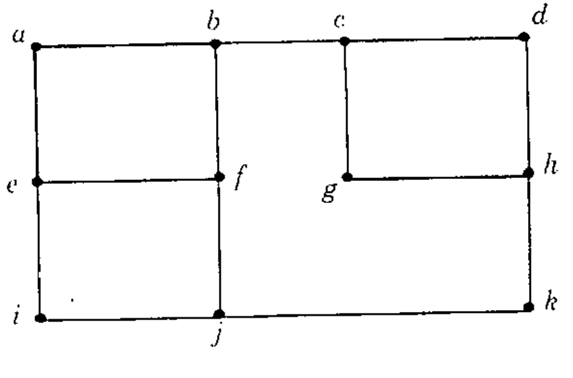
6. (a) (i) Find a matching or explain why none exists for the following graph : 3



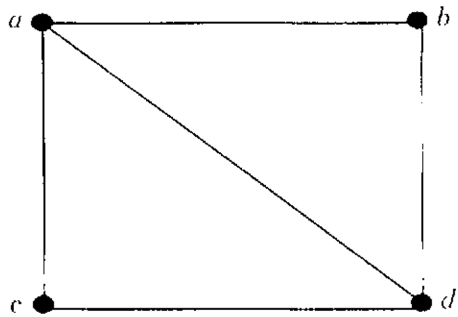
(*ii*) Find a vertex basis for the following graph : 3



(*b*) (*i*) Find a maximal independent set of the following graph : 3



(*ii*) Is the following graph an interval graph ? If yes, give a model of overlapping intervals : 3



(c) Suppose we are given three pitchers of water of sizes $10l$, $7l$, $4l$. Initially $10l$ pitcher is full and other two are empty. We pour water from one pitcher into another till the receiving pitcher is full or the pouring pitcher is empty. Find a minimal sequence of pouring $2l$ water in either of $7l$ or $4l$ pitcher. 6