

This question paper contains 4+2 printed pages]

Your Roll No.....

7562

**B.A. (Programme)/II D-I**

**MATHEMATICS—Paper II**

(Geometry, Differential Equations and Algebra)

(NC—Admission of 2004 onwards)

Time : 3 Hours

Maximum Marks : 100

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each question.

1. (a) Find an equation of the parabola that is symmetric about the  $y$ -axis, has its vertex at the origin and passes through the point (5, 2). 8

- (b) Describe the graph of the equation : 8

$$x^2 - y^2 - 4x + 8y - 21 = 0.$$

- (c) Rotate the coordinate axes to remove the  $xy$  term. Then identify the type of conic and sketch its graph : 8

$$25x^2 - 14xy + 25y^2 - 288 = 0.$$

P.T.O.

2. (a) Find equations of two spheres that are centered at the origin and are tangent to the sphere of radius 1 centered at  $(0, 0, 7)$ . 8.5

- (b) Show that the lines :

$$x = -2 + t, y = 3 + 2t, z = 4 - t$$

$$x = 3 - t, y = 4 - 2t, z = t$$

are parallel and find an equation of the plane they determine. 8.5

- (c) Find the area of the triangle that is determined by the points  $p_1(2, 2, 0)$ ,  $p_2(-1, 0, 2)$ , and  $p_3(0, 4, 3)$ . 8.5

3. (a) (i) Solve  $dx/dt + 5x + y = e^t$ .

$$dy/dt - x + 3y = e^{2t}$$

(ii) Solve :

$$(D^3 - 7D - 6)y = e^{2x}(1 + x). \quad 10$$

(b) Using the method of variation of parameters,

solve :

$$(D^2 - 1)y = 2/(1 + e^x). \quad 10$$

(c) A large tank initially contains 50 gal of brine in which

there is dissolved 10 lb of salt brine containing 2 lb

of dissolved salt per gallon flows into the tank at the

rate of 5 gal/min. The mixture is kept uniform by stirring

and the stirred mixture simultaneously flows out at the

slower rate of 3 gal/min. How much salt is in the tank

at any time  $t > 0$  ?

10

4. (a) Find the partial differential equation by eliminating  $a, b, c$  from

$$x^2/a^2 + y^2/b^2 + z^2/c^2 = 1. \quad 6.5$$

- (b) Solve :

$$z(p - q) = z^2 + (x + y)^2. \quad 6.5$$

- (c) Find the complete integral of  $q = px + p^2$ . 6.5

5. (a) Define a group. Prove that the set  $I$  of all the integers with binary operation  $*$  defined by  $a * b = a + b + 1$   $a, b \in I$ , is an abelian group. 8.5

- (b) Write

$$\left( \begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 8 & 2 & 10 & 11 & 5 & 9 & 4 & 6 & 1 & 3 & 12 & 7 \end{array} \right)$$

3.

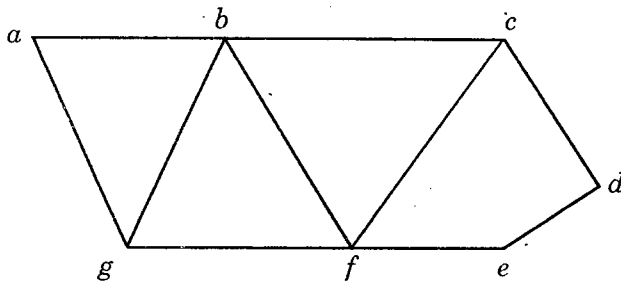
As a product of disjoint cycles and also construct the associated diagram. 8.5

- (c) Discuss the clockwise rotations of a square and list all the permutations obtained by the clockwise rotations. 8.5

6. (i) Solve the following travelling salesman problem : 8.5

To		1	2	3	4
From	1	—	7	2	6
	2	1	—	8	9
	3	2	1	—	3
	4	4	2	6	—

- (ii) For the following graph, find all sets of 2 vertices whose removal disconnects the graph of remaining vertices. 8.5



(iii) Suppose we are given three pitchers of water of sizes 10 litres, 7 litres and 4 litres. Initially the 10 litres pitcher is full and the other two empty. We can pour water from one pitcher into another, pouring until the receiving pitcher is full or the pouring pitcher is empty. Is there a way to pour among pitchers to obtain exactly 2 litres in the 7 or 4 litres pitcher? If so, find a minimal sequence of pouring to get 2 quarts in the 7-litre or 4-litre pitcher.

8.5