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Your Roll No.....

7563

B.A. Programme/II

D-I

MATHEMATICS—Paper II

(Geometry, Differential Equations and Algebra)

(NC—Admissions of 2004 onwards)

Time : 3 Hours

Maximum Marks : 100

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (a) Sketch the graph of the hyperbola :

$$y^2 - x^2 = 1$$

showing their vertices, foci and asymptotes. 8

- (b) State and explain reflection properties of parabola, ellipse and hyperbola with the help of sketches. 8

- (c) Find an equation for the ellipse that satisfies the given conditions, length of major axis 26, foci $(\pm 5, 0)$. 8

P.T.O.

2. (a) Show that the lines L_1 and L_2 are skew :

$$L_1 : x = 1 + 7t, y = 3 + t, z = 5 - 3t$$

$$L_2 : x = 4 - t, y = 6, z = 7 + 2t \quad 8.5$$

- (b) Find an equation for the line L of intersection of the planes :

$$2x - 4y + 4z = 6$$

and $6x + 2y - 3z = 4. \quad 8.5$

- (c) Show that planes :

$$x + 2y - 2z = 3$$

and $2x + 4y - 4z = 7$

are parallel.

Find the distance between the planes. 8.5

3. (a) (i) Solve :

$$(D^2 + 3D + 2)y = e^{2x} \sin x.$$

(ii) Solve :

$$x \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + 2y = x \log x. \quad 10$$

(b) (i) Solve :

$$\frac{d^2 y}{dx^2} + y = \operatorname{cosec} x,$$

by the method of variation of parameters.

(ii) Solve :

$$(x^2 + y^2 + 2x)dx + 2ydy = 0. \quad 10$$

(c) (i) Solve :

$$\frac{adx}{(b-c)yz} = \frac{bdy}{(c-a)zx} = \frac{cdz}{(a-b)xy}.$$

(ii) Solve :

$$(x^2 + y^2 + z^2)dx - 2xydy - 2xzdz = 0. \quad 10$$

4. (a) (i) Eliminate the arbitrary function f from the equation :

$$x + y + z = f(x^2 + y^2 + z^2).$$

- (ii) Find the general solution of the differential equation :

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z. \quad 6.5$$

- (b) (i) Find the general integral of the linear partial differential equation :

$$px(x + y) = qy(x + y) - (x - y)(2x + 2y + z).$$

- (ii) Find the complete integral of the equation :

$$p = (z + qy)^2. \quad 6.5$$

- (c) (i) Find whether the equation :

$$y(x + y)(r - s) - xp - yq - z = 0$$

is elliptic, parabolic or hyperbolic.

- (ii) Solve : 6.5

$$p + q = pq.$$

5. (a) Prove that the set $\{1, 2, 3, 4, 5, 6\}$ is a finite abelian group of order 6 under multiplication modulo 7. 8.5

(b) Write :

$$\left(\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 9 & 8 & 7 & 3 & 4 & 6 & 1 & 2 \end{array} \right).$$

as product of disjoint cycles and as a product of a transposition. Also construct the associated diagram. 8.5

- (c) Discuss the flip motion of a square and also list all the permutations obtained by flip motion of a square. 8.5

6. (a) Solve a travelling salesman problem for the given cost matrix :

To	1	2	3	4	
1	∞	3	9	7	
From 2	3	∞	6	5	
3	8	6	∞	6	
4	9	7	4	∞	8.5

P.T.O.

- (b) A smoking machine is used to test the tar-content of four brands of cigarettes; the machine has four parts so that four cigarettes can be smoked simultaneously. However, these four parts may not be identical and that might affect the measurement of the tar-content. Also if four runs were made on the machine testing one brand at a time, the humidity could change, thus affecting the results. Show how to reduce the errors due to the different parts and the different runs by using a Latin square to design the experiment. 8.5
- (c) Find a matching or explain why none exists for the following graph : 8.5

