This question paper contains 8+2 printed pages]

Your Roll No.................

5512

B.A. (Programme)/II **D**

MATHEMATICS—Paper II

(Geometry, Differential Equations and Algebra)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :— The maximum marks printed on the question paper are applicable for the students of regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

All questions are compulsory.

Attempt any *two* parts from each question.

P.T.O.

1. (*a*) Identify and sketch the curve : 6

$$xy = 1$$

(*b*) Find the equation of the ellipse and sketch the curve :

$$5x^2 + 9y^2 - 20x + 54y = -56$$

Also label the foci, the vertices and the ends of the minor axis. 6

(*c*) (*i*) Find an equation of a parabola whose vertex is $(5, -3)$, axis parallel to y-axis and parabola passes through $(9, 5)$.

(*ii*) Find the equation for a hyperbola whose vertice are $(\pm 1, 0)$ and asymptotes are $y = \pm 2x$. 3+3

2. (*a*) A sphere S has centre in the first octant and is tangent to each of the three coordinate planes. The distance from the origin to the sphere is $3 - \sqrt{3}$ units. What is the equation of the sphere ? 6½

(*b*) (*i*) Find the equation of the plane through the point $(-1, 2, -5)$ and perpenlicular to planes $2x - y + z = 1$ and $x + y - 2z = 3$.

(*ii*) Find the distance between the point $(1, -2, 3)$ and the plane $2x - 2y + z = 4$ 3½+3

(*c*) (*i*) Find the parametric equation of the line L passing through the points $(2, 4, -1)$ and $(5, 0, 7)$, where does the lines intersect the *xy*-plane ?

(*ii*) Determine whether the line :

$x = 3 + 8t$

$y = 4 + 5t$

$z = -3 - t$

is parallel to the plane $x - 3y + 5z = 12$. 3½+3

3. (*a*) (*i*) Solve the differential equation :

$$[y^2(x+1)+y]dx+(2xy+1)dy=0$$

by finding an integrating factor.

(*ii*) Solve :

$$y=2px-xp^2$$

where

$$p=\frac{dy}{dx}$$ 3+3

(*b*) (*i*) Solve the equation :

$$(D^2+4)y=\sin 3x+e^x+x^2$$

(*ii*) Solve the equation :

$$y''+4y=4\tan 2x$$

by the method of variation of parameters. 3+3 (*c*) (*i*) Solve :

$$\frac{dx}{dt} + 4y = \sec^2 2t$$

$$\frac{dy}{dt} = x$$

(*ii*) Using the concept of Wronskian show that e^{2x} and e^{3x} are linearly independent solution of the differential equation $y'' - 5y' + 6y = 0$ where $y' = \frac{dy}{dx}$.

Find the general solution $y(x)$ satisfying the conditions $y(0) = 0$ and $y'(0) = 1$. 3+3

4. (*a*) (*i*) From the partial differential equation by eliminating the arbitrary frunction f from the relation :

$$x + y + z = f(x^2 + y^2 + z^2)$$

(*ii*) Find whether the equation :

$$y^2 r - 2xys + x^2 t = \frac{y^2}{x} p + \frac{x^2}{y} q$$

is hyperbolic, parabolic or elliptic. 4+2½

(*b*) (*i*) Find the general integral of the linear partial differential equation :

$$(y + z)p + (z + x)q = x + y$$

(*ii*) Find the complete integral of the equation :

$$p + q = pq$$ 4½+2

(*c*) (*i*) Find a complete integral of :

$$(p + q)(px + qy) = 1$$

(*ii*) Find the general solution of : 4+2½

$$x^2 p + y^2 q = (x + y)z$$

5. (*a*) (*i*) State Lagrange's theorem. Hence show that if G is finite group then $a^{0(G)} = e \forall a \in G$.

(*ii*) If A and B are subrings of a ring R, show that $A \cap B$ is also a subring of R. 3+3

(*b*) (*i*) Show that the set I of all integers with binary operation defined as :

$$a * b = a + b - 1 \forall a, b \in I.$$

(*ii*) Write :

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 9 & 5 & 4 & 7 & 8 & 1 & 6 & 10 & 2 \end{pmatrix}$$

(*a*) a product of disjoint cycles;

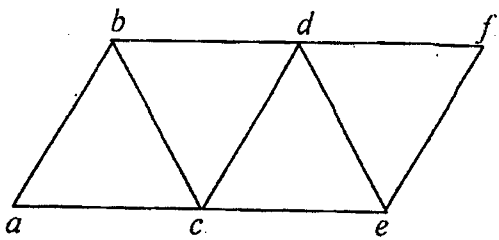
(*b*) product of transpositions.

Find the inverse and order of σ. 3+3 (*c*) (*i*) Prove that the intersection of any collection of subgroups of a group is again a subgroup.

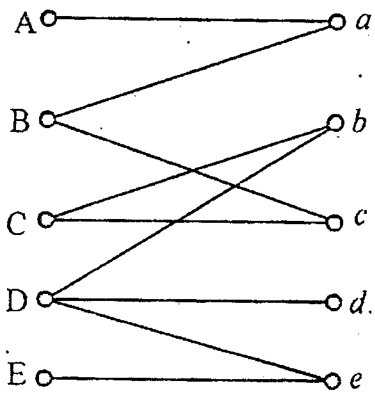
(*ii*) Let G be a group in which $(ab)^2 = a^2b^2$ $\forall a, c \in G$. Prove that G is abelian. 3+3½

6. (*a*) (*i*) Define a Latin square. Give an example of a Latin square of order 3. Is it unique ? Justify.

(*ii*) Is the following graph an interval graph ? If yes, give a model of overlapping intervals, if not, explain why not ? 3½+3



(*b*) (*i*) Find a matching or explain why none exists for the following graph :



(*ii*) Three pitchers of sizes 8 liters, 5 liters and 3 liters are given. If initially 8 liter's pitcher is full of water and the other two empty, find a minimal sequence of pouring so as to have exactly 4 liters of water in one pitcher. 3+3½ (*c*) Solve the travelling salesperson problem for the cost matrix : 6½

To		1	2	3	4
From	1	—	7	2	6
	2	1	—	8	9
	3	2	1	—	3
	4	4	2	6	—