This question paper contains 8 printed pages]

Your Roll No.....

5513

B.A. (Programme.)/II **D**

MATHEMATICS—Paper II

(Geometry, Differential Equations and Algebra)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :— The maximum marks printed on the question paper are applicable for the students of regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

All questions are compulsory.

Attempt any *two* parts from each question.

P.T.O.

1. (*a*) Describe and sketch the conic

$$(x + 3)^2 + 4(y - 5)^2 = 16$$

and also label the foci, vertices and end of minor axis. 6

(*b*) Identify and sketch the curve :

$$x^2 - 4y^2 + 2x + 8y - 7 = 0.$$

Also label the vertices, foci and asymptotes. 6

(*c*) Sketch the parabola :

$$y^2 - 6y - 2x + 1 = 0$$

and label the focus, vertex and directrix. Also find the length of latus rectum. 6

2. (*a*) Find the equations of *two* spheres that are centered at the origin and are tangent to the sphere of radius 1 centered at (0, 0, 7). 6½

(*b*) (*i*) Express the vector $\vec{v}$ as the sum of a vector parallel to $\vec{b}$ and a vector orthogonal to $\vec{b}$, where $\vec{v} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{k}$.

(*ii*) If

$$\vec{u} = \hat{i} + 3\hat{j} - \hat{k}$$

$$\vec{v} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{w} = 3\hat{i} - \hat{j} + 2\hat{k}$$

find :

$\vec{u} \times (\vec{v} \times \vec{w})$ and $(\vec{u} \times \vec{v}) \times \vec{w}$. 4+2½

(*c*) Show that the lines :

$L_1 : x = -2 + t, y = 3 + 2t, z = 4 - t; -\infty < t < \infty$ and

$L_2 : x = 3 - t, y = 4 - 2t, z = t; -\infty < t < \infty$

are parallel and the equation of the plane they determine. 6½

P.T.O.

3. (*a*) (*i*) Solve :

$$x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = x^3$$

(*ii*) Solve :

$$(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0. \qquad 6\frac{1}{2}$$

(*b*) Solve the equation :

$$\frac{d^2y}{dx^2} + 4y = \sin^2 2x,$$

by the method of variation of parameters. $6\frac{1}{2}$

(*c*) (*i*) Show that $e^x \sin x$ and $e^x \cos x$ are linearly independent solution of :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

Also find the solution if :

$$y(0) = 2 \text{ and } y'(0) = 3.$$

(*ii*) Find the orthogonal trajectories of the family of curves

$$x^2 + y^2 = Cx^3$$

where C is an arbitrary constants. $4+2\frac{1}{2}$

4. (*a*) (*i*) Find the general integral of the partial differential equation :

$$px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3)$$

(*ii*) Find whether the equation :

$$x^2r - y^2t - px - qy = x^2$$

is hyperbolic, parabolic or elliptic. 4+2

(*b*) (*i*) Find the complete integral of the equation :

$$p^2z^2 + q^2 = 1.$$

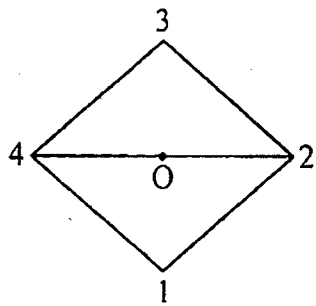
(*ii*) Eliminate the arbitrary function F from the equation :

$$z = x + y + f(xy).$$ 4+2

(*c*) Find a complete integral of $(p + q)(px + qy) = 1$. 6

5. (*a*) Let $a = (1\ 2\ 3\ 4)$ represent the counterclockwise rotation by 90° about its centre O of a square, whose vertices are marked 1, 2, 3, 4.

Express (13) (24) and (1432) as powers of a and determine what they represent geometrically . 6½



(*b*) State Lagrange's theorem for groups and use it to prove that if 'a' is any positive relatively prime to the positive integers n, then 6½

$$a^{\phi(n)} \equiv \text{I}(\text{mod } n),$$

where (ϕ) is the Euler's ϕ function.

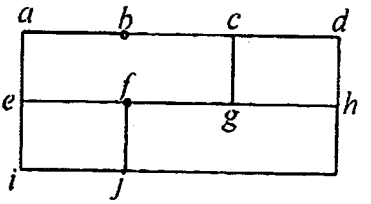
(*c*) (*i*) If (R, +) is a ring and $a, b, c, d \in$ R, then prove that :

$$(a - b)(c - d) = (ac + bd) - (ad + bc)$$

and $(a - b)(c + d) = (ac + ad) - (bc + bd)$.

(*ii*) Let H be a non-empty subset of a group G. Prove that H is a subgroup of G iff $ab^{-1} \in H$ for all $a, b \in H$. 6½

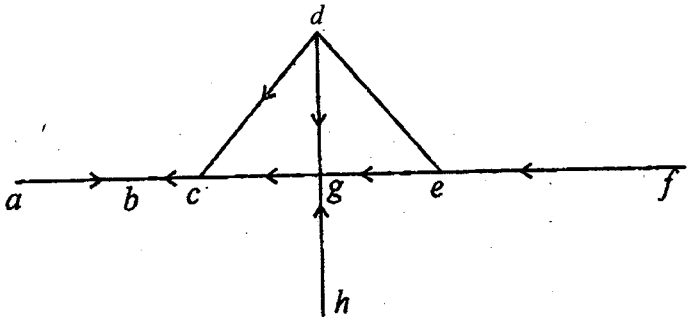
6. (*a*) Find a minimal edge cover for the following graph. Give a detailed logical analysis : 6



(*b*) Solve the travelling salesperson problem for the cost matrix : 6

To	1	2	3	4	
From	1	—	7	2	6
	2	1	—	8	9
	3	2	1	—	3
	4	4	2	6	—

(*c*) (*i*) Find a vertex basis for the following graph. Justify your answer.



(*ii*) Find all possible matchings or explain why none exists : 3+3