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Your Roll No. ....

6736

B.A./B.Sc. (Hons.)/II **D**

MATHEMATICS—Unit VI

(Differential Equations—I)

(Admissions of 2008 and before)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *All* the Sections.

All questions carry equal marks.

Section I

1. Solve any *two* of the following :

(i) $\frac{1}{y}\frac{dy}{dx} + \frac{x}{1-x^2} = xy^{-\frac{1}{2}}$

(ii) $x\,dy - \{y + xy^3(1 + \log x)\}\,dx = 0$

(iii) $x^2p^2 - 2xpy + y^2 - x^2y^2 - x^4 = 2px -$

P.T.O.

2. In a certain city the population gets double in 2 years and after 3 years the population is 20,000. Find the number of people initially living in the city.

Section II

3. If $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of the differential equation :

$$a_0(x)\, y'' + a_1(x) y' + a_2(x) y = 0\,,$$

then prove that every solution other solution of the equation is a linear combination of the two solutions $y_1(x)$ and $y_2(x)$. Hence, show that every solution of $y'' + y = 0$ is a linear combination of :

$$\cos x + \sin x \text{ and } \cos x - \sin x.$$

4. Solve any *two* of the following :

(*i*) $\left(D^4 + 2D^2 + 1\right) y = x^2 \cos x,$ where $D = \dfrac{d}{dx}$

(*ii*) $y'' + 2y = x^2 e^{3x} + x^2 \cos 2x.$

(*iii*) Solve using variation of parameters :

$$x^2 y'' - 3xy' + 3y = 2x^5 .$$

Section III

5. Find the power series solution of any *two* of the following :

(*a*) $2x^2 y'' - xy' - (x-5)\,y = 0$ about $x_0 = 0$

(*b*) $x^2 y'' + xy' - (x^2 - \frac{1}{4})y = 0$ about $x_0 = 0$

(*c*) $8x^3 y'' + 2x^2 y' + y = 0$ about $x_0 = \infty$ where $x > 0$.

Section IV

6. Solve any *two* of the following :

(*a*) $\frac{dy}{dt} - 3x + 4y = e^{-2t}$: $\frac{dy}{dt} - x + 2y = 3e^{-2t}$

(b) $$\frac{dx}{y^2 + yz + z^2} = \frac{dy}{z^2 + zx + x^2} = \frac{dz}{x^2 + xy + y^2}$$

(c) $$(x^2y - y^3 - y^2z)\,dx + (y^2x - x^2z - x^3)\,dy + (x^2y + y^2x)\,dz = 0$$

7. Using Picard's method find upto three successive approximations, the solution of the differential equation :

$$\frac{dy}{dx} = e^x + e^y, \; y(0) = 0.$$