This question paper contains 4 printed pages]

Your Roll No.................

6740

B.A./B.Sc. (Hons.)/II **D**

MATHEMATICS—Unit X

(Probability and Mathematical Statistics)

(Admissions of 2008 and before)

Time : 2 Hours *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

Section I

1. (*a*) State and prove Bayes' theorem. 4½

(*b*) Two unbiased dice are thrown. If X is the sum of the numbers showing up, then prove that :

$$P\{|X - 7| \geq 3\} \leq \frac{35}{54}.$$

Compare it with the actual probability. 4½

(c) Let

$$f(x) = Ke^{-ax}\ (1 - e^{-ax})\, I_{(0,\ \infty)}(x).$$

(i) Find K such that $f(.)$ is a density function.

(ii) Find the cumulative distribution function. 4½

Section II

2. (a) Find the moment generating function, mean and variance of gamma distribution. Is exponential distribution a particular case of gamma distribution ? Justify your answer. 5

(b) (i) Show that in a Poisson distribution with unit mean, mean deviation about mean is $(2/e)$ times the standard deviation.

(ii) If $E[X] = 10$ and $\sigma_x = 3$, can X have a negative binomial distribution ? Justify your answer. 5

(c) Show that for normal distribution mean, median, mode coincide. 5

Section III

3. (*a*) Let

$$f(x,\ y) = xy\ I_{(0,1)}(x)\, I_{(0,x)}(y).$$

(*i*) Find marginal density functions of X and Y.

(*ii*) Find P{0 < X + Y < 1}. 5

(*b*) Three fair coins are tossed. Let X denote the number of heads on the first two coins and let Y denote the number of tails on the last two coins :

(*i*) Find the joint distribution of X and Y.

(*ii*) Find Cov[X, Y].

(*iii*) Find $f_{Y/X}$ (Y|X = 2). 5

(*c*) Define the joint cumulative distribution function of a two dimensional random variable (X, Y) and state its properties. Further show that :

$$F_Y(x) + F_Y(y) - 1 \le F_{X,Y}(x, y) \le \sqrt{F_X(x) F_Y(y)},\ \forall\, x, y,$$

where F(.) denotes the cumulative distribution function. 5

Section IV

4. (*a*) State Weak Law of Large numbers. Suppose that some distribution with an unknown mean has variance equal to 1. How large a sample must be taken in order that the probability will be at least 0.95 that the sample mean $\bar{X}_n$ will lie within 0.5 of the population mean ? 4½

(*b*) Find the moment generating function and characteristic function of the distribution : 4½

$$f(x) = \begin{cases} 1 - |x|, & |x| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(*c*) Use central limit theorem to show that if X is binomially distributed with parameters *n* and *p*, then : 4½

$$Z = \frac{X - np}{\sqrt{npq}} \to N(0, 1) \text{ as } n \to \infty.$$