[This question paper contains 6 printed pages.]

1404 *Your Roll No.* ....

B.A. Programme/II **E-I**

MATHEMATICS–Paper II

(Geometry, Differential Equations and Algebra)

(NC–Admission of 2004 onwards)

Time : 3 Hours *Maximum Marks : 100*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.
*Attempt any **two** parts from each question.*

1. (a) Identify and sketch the curve

$x^2 - 4x + 2y = 1.$ (8)

(b) Sketch the ellipse

$9x^2 + 4y^2 + 18x - 24y + 9 = 0$

and also label the foci, the vertices and the ends of the major axis. (8)

P.T.O.

(c) Find an equation for the hyperbola that passes through the origin and whose asymptotes are $y = 2x + 1$ and $y = -2x + 3$. (8)

2. (a) A sphere S has center in the first octant and is tangent to each of the three co-ordinate planes. The distance from the origin to the sphere is $3 - \sqrt{3}$ units. What is the equation of the sphere? (8½)

(b) (i) Find the vector of length 2 that makes an angle $\frac{\pi}{4}$ with the positive x-axis.

(ii) Find the angle between the vectors $\mathbf{u} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\mathbf{v} = -3\hat{i} + 6\hat{j} + 2\hat{k}$. (4,4½)

(c) (i) Find an equation of the plane passing through the point $P(-3, 0, 7)$ and perpendicular to the vector $\mathbf{n} = 5\hat{i} + 2\hat{j} - \hat{k}$.

(ii) Determine whether the line

$x = 4 + 2t \quad y = -t \quad z = -1 - 4t$

is parallel or perpendicular to the plane $3x + 2y + z - 7 = 0$. (4,4½)

3. (a) Solve the differential equation

$$y'' + y = \sec x$$

by the method of variation of parameters. (8½)

(b) Solve the equation

$$(yz + 2x)\, dx + (zx - 2z)\, dy + (xy - 2y)\, dz = 0$$

OR

Find the orthogonal trajectories of the family of parabolas

$3xy = x^3 - a^3$, 'a' being parameter of family. (8½)

(c) Solve the differential equation

$$(D^2 - 4)\, y = e^x + \sin x$$ (8½)

4. (a) Find the complete integral of the partial differential equation

$$p^2 + q^2 = 4$$ (8)

(b) (i) Find the general integral of the partial differential equation

$$p \tan x + q \tan y = \tan z$$

(ii) Find whether the equation

$$\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=0$$

is hyperbolic, parabolic or elliptic. (6,2)

(c) Find the complete integral of

$z = px + qy + p^2 + q^2$ (8)

5. (a) Express the following permutation as a product of disjoint cycles

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 3 & 2 & 7 & 1 & 4 & 6 \end{pmatrix}$$ (8½)

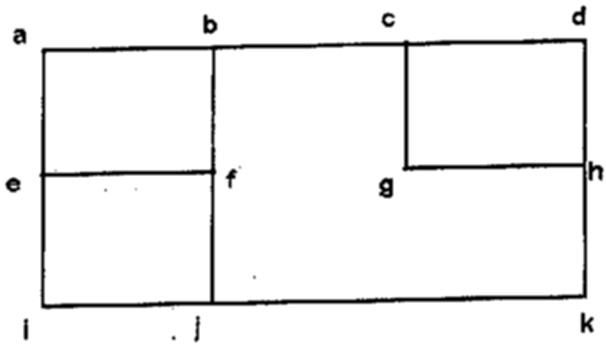
(b) Let G be a group and H be a nonempty subset of G. Then H is a subgroup of G if and only if $ab^{-1} \in H \; \forall \; a, b \in H$. (8½)

(c) Show that the set of all matrices of the form

$$\begin{pmatrix} x & y \\ -y & x \end{pmatrix}; \; x, y \in \mathbf{Z}$$

is a ring with respect to matrix addition and matrix multiplication. (8½)

6. (a) (i) The following figure represents a section of city's street map. We want to position police at corners (vertices) so that they can keep every block (edge) under surveillance i.e. every edge should have a policeman at least one of its corner. What is the smallest number of police that can do this job?



(ii) Show that the given Latin square can not be obtained from a group table:

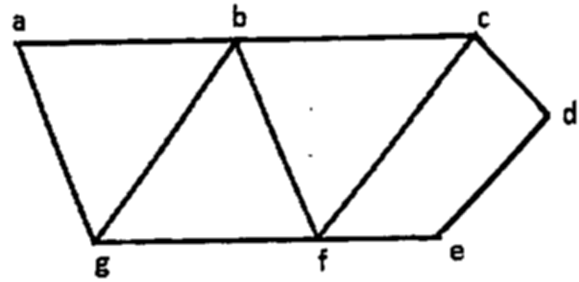
A	B	C	D	E
B	A	E	C	D
C	D	A	E	B
D	E	B	A	C
E	C	D	B	A

(8)

(b) In the following figure find

(i) All sets of two vertices whose removal disconnects the graph.

(ii) All sets of two edges whose removal disconnects the graph. (8)



(c) The following is the cost matrix for the traveling salesperson problem. Should C_{23} be used? Justify.

	To	1	2	3	4
	1	-	3	9	7
From	2	3	-	6	5
	3	5	6	-	6
	4	9	7	4	-

(8)