[This question paper contains 6 printed pages.]

1403 *Your Roll No.* ....

B.A. Programme/II **E-I**

MATHEMATICS–Paper II

(Geometry, Differential Equations and Algebra)

(NC–Admission of 2004 onwards)

Time : 3 Hours *Maximum Marks : 100*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All Questions are compulsory.
Attempt any ***two*** *parts from each question.*

1. (a) Find the center, vertices, foci and asymptotes of the hyperbola whose equation is

$$4y^2 - x^2 + 40y - 4x + 60 = 0$$

and sketch its graph. (8)

(b) Identify and sketch the curve

$$y^2 - 8x - 6y - 23 = 0$$

and also label the focus, vertex and directrix. (8)

(c) Find an equation for the ellipse with length of major axis 6 and with foci (2, 1) and (2, –3). (8)

2. (a) Find the equation of the sphere that has (1, –2, 4) and (3, 4, –12) as end points of a diameter. (8½)

(b) (i) Find the angle that the vector $\mathbf{v} = -\sqrt{3}\,\hat{i} + \hat{j}$ makes with the positive x-axis.

(ii) Use a scalar triple product to determine whether the vectors lie in the same plane

$\mathbf{u} = 5\hat{i} - 2\hat{j} + \hat{k}$, $\mathbf{v} = 4\hat{i} - \hat{j} + \hat{k}$,

$\mathbf{w} = \hat{i} - \hat{j}$ (4,4½)

(c) (i) Find the parametric equations of the line L passing through the points P(2, 4, –1) and Q(5, 0, 7).

(ii) Find the equation of the plane through the points (1, 2, –1), (2, 3, 1) and (3, –1, 2). (4,4½)

3. (a) Solve the differential equation

$$y'' - 2y' = e^x \sin x$$

by the method of variation of parameters. (8½)

(b) Check the condition of integrability of the following differential equation and hence solve the equation

$$(yz + xyz)\,dx + (zx + xyz)\,dy + (xy + xyz)\,dz = 0$$

(8½)

(c) Show that $e^x \sin x$ and $e^x \cos x$ are linearly independent solutions of the equation

$$y'' - 2y' + 2y = 0.$$

What is the general solution? Also find the solution $y(x)$ with the property that $y(0) = 2$, $y'(0) = 3$.

(8½)

4. (a) Find the complete integral of the partial differential equation

$$p^2 - y^2 q = y^2 - x^2.$$ (8½)

(b) (i) Find the general integral of the partial differential equation

$$(2xy - 1)\,p + (z - 2x^2)\,q = 2\,(x - yz)$$

(ii) Find the nature (elliptic, hyperbolic or parabolic) of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} = (1+y)^2 \frac{\partial^2 z}{\partial y^2}.$$ (6,2½)

(c) Find the complete integral of

$pxy + pq + qy = yz$ (8½)

5. (a) For the permutations

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 4 & 1 & 7 & 5 & 6 \end{pmatrix}$$

and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 1 & 6 & 7 & 4 & 5 \end{pmatrix}$

Calculate $\sigma\tau\sigma^{-1}$. (8½)

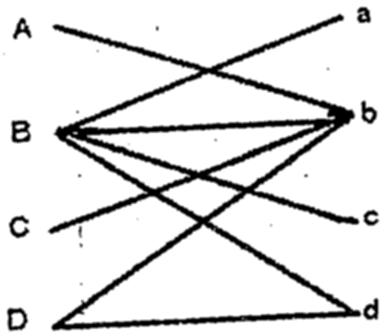
(b) Let G be a group and H be a subgroup of G. Prove that the order of the subgroup H of G divides the order of the group G. (8½)

(c) If (R, +, .) is a ring in which every nonzero element has a multiplicative inverse, then show that for all a, b $\in$ R

$a.b = 0 \Rightarrow a = 0$ or $b = 0$ (8½)

6. (a) (i) Construct a Latin square of order 5 on {0,1,2,3, 4}.

(ii) Find a matching or explain why none exists for the following graph:

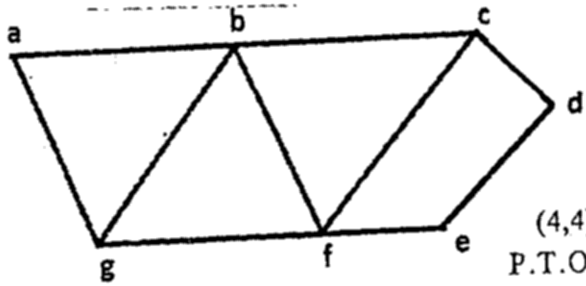


(4,4)

(b) In the following figure find

(i) All sets of two vertices whose removal disconnects the graph.

(ii) All sets of two edges whose removal disconnects the graph.



(4,4)

P.T.O.

(c) The following is the cost matrix for the traveling salesperson problem. Should C_{23} be used? Justify.

	To	1	2	3	4
	1	-	7	1	3
From	2	7	-	4	5
	3	5	4	-	4
	4	1	3	6	-

(8)