[This question paper contains 6 printed pages.]

335 *Your Roll No.* ....

B.A. (Programme) / II **E**

MATHEMATICS – Paper II

(Geometry, Differential Equations and Algebra)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :– The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. A). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

All *questions are compulsory.*
Attempt any **Two** *parts from each questions.*

1. (a) Sketch the hyper-bola

$$4(y-3)^2 - 9(x-2)^2 = 36$$

Also label the vertices, foci and asymptotes. (4+2)

(b) Identify and sketch the curve

$$x^2 + 9y^2 + 2x - 18y + 1 = 0$$

and also label the foci, the vertices and the ends of the miror axis. (6)

(c) Find the equations for the parabola which passes through the point (3,2) and $\left(2, -\sqrt{2}\right)$ and whose axis is $y = 0$ and sketch it.

By drawing the tangent at (3,2), illustrate the property of reflection of parabola. (4+2)

2. (a) Show that the equation of the sphere passing through the three points (3,0,2), (–1,1,1), (2,–5,4) and having its centre on the plane $2x + 3y + 4z = 6$ is $x^2 + y^2 + z^2 + 4y - 6z = 1$. (6½)

(b) (i) Find the equation of the plane through points (1, 2, –1), (2, 3, 1) and (3, –1, 2).

(ii) Determine whether the vectors.

$\vec{u} = (1, 2, -1)$, $\vec{v} = (3, 0, -2)$ and $\vec{w} = (5, -4, 0)$.

lie in the same plane ? Also find $\vec{u} \times \vec{v}$ and show that it is orthogonal to $\vec{u}$. (3½+3)

(c) Show that the lines

$L_1 : x = 1 + 7t,\ y = 3 + t,\ z = 5 - 3t;\ -\infty < t < \infty$

$L_2 : x = 4 - t,\ y = 6,\ z = 7 + 2t;\ -\infty < t < \infty$

are skew lines and find the distance between them. (6½)

3. (a) (i) Solve : $(x^2 + y^2 + x)dx + xy\,dy = 0$

(ii) Solve : $(D^2 + 3D + 2)y = e^{2x}\sin x$ (3+3½)

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x + \cos x$$

by the method of variation of parameters. (6½)

(c) In a certain bacteria culture the rate of increase in the number of bacteria is proportional to the number of bacteria present. It is given that the number triples in 5 hours.

(i) Formulate the problem mathematically and find the number of bacteria that will be present in 10 hours.

(ii) Also determine the time when the number present will be 10 times the number initially present. (6½)

4. (a) (i) Find a complete integral of $p = (z + qy)^2$.

(ii) Classify the following partial differential equation into elliptic, parabola or hyperbola and find its order and degree as well.

$$x(y-x)r-(y^2-x^2)s+y(y-x)t+(y+x)(p-q)=0$$

(4+2)

(b) (i) Find the general solution of the partial differential equation

$$(y+z)p + (z+x)q = x+y$$

(ii) Eliminate the constants a and b from the equation

$$ax^2 + by^2 + cz^2 = 1$$ (4+2)

(c) Find the complete integral of the partial differential equation

$$2(z+px+qy) = yp^2$$ (6)

5. (a) (i) Let $\sigma \in S_n$ be written as a product of disjoint cycles. Show that the order of σ is the L.C.M. of the lengths of its cycles.

(ii) Let $G = R - \{1\}$, the set of all real numbers excluding 1. If a binary operation * on G is defined by

$$a * b = a + b - ab$$

Show that (G,*) is a group. (3½+3)

(b) Show that the set Z_7, of integers modulo 7 forms a commutative ring under addition and multiplication of congruence classes (i.e., addition and multiplication modulo 7). (6½)

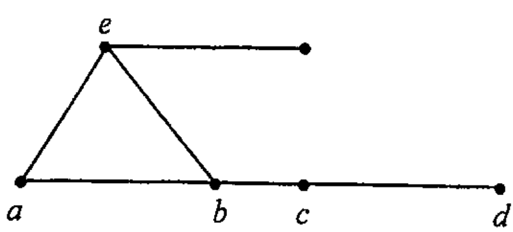
(c) (i) Show that every group of order 21 is cyclic.

(ii) If H and K are subgroups of a group G, show that $H \cap K$ is also a subgroup of G.

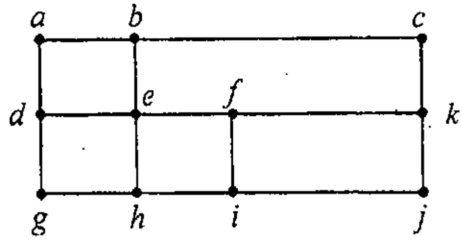
(3½+3)

6. (a) (i) Define a Latin square. Give an example of a Latin square of order 6.

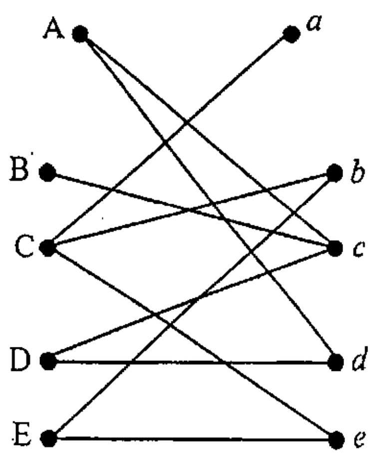
(ii) Is the following graph an interval graph ? Justify. (3+3)



(b) (i) Find a maximum independent set for the following graph. What is the minimum number of independent sets needed to cover all the vertices ?



(ii) Find a matching or explain why none exist from the following graph :



(3+3)

(c) Solve the travelling sales person problem for which the cost matrix is given by the table :

	To	1	2	3	4
From	1	—	3	9	7
	2	4	—	5	6
	3	5	2	—	3
	4	6	7	4	—

(6)