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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **374**

Unique Paper Code : **235351** **C**

Name of the Paper : **Paper A : Integration and Differential Equations (Admissions of 2011 and onwards)**

Name of the Course : **B.A. (Prog.) Mathematics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Show that :

$$\int_0^{\frac{\pi}{2}} \log(\sin x)\, dx = \frac{\pi}{2} \log \frac{1}{2}.$$ 6

(*b*) If

$$U_n = \int_0^{\frac{\pi}{2}} x^n \sin x \, dx, \ (n > 1)$$

then prove that :

$$U_n + n(n-1)U_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}.$$ 6

(*c*) Find the area of the smaller portion enclosed by the curves :

$$x^2 + y^2 = 16;\ y^2 = 6x.$$ 6

P.T.O.

2. (*a*) Find the volume of the solid generated by rotating the ellipse :

$$9x^2 + 4y^2 = 36 \text{ about } x\text{-axis.}$$ 6½

(*b*) Find the length of the arc of the parabola $x^2 = 4ay$ measured from the vertex to one extremity of the latus rectum. 6½

(*c*) Evaluate :

(*i*) $$\int \frac{dx}{(2-x)\sqrt{1-2x+3x^2}}$$

(*ii*) $$\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}.$$ 3,3½

3. (*a*) Show that :

$$\int_0^{\frac{\pi}{2}} \cos^m x \cos nx\,dx = \frac{m}{m+n}\int_0^{\frac{\pi}{2}} \cos^{m-1} x \cos(n-1)x\,dx.$$ 6

(*b*) Solve :

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = x^3 + x + e^{-2x}.$$ 6

(*c*) Solve :

$$y = 2px - xp^2, \text{ where } p = \frac{dy}{dx}.$$ 6

4. (*a*) Solve :

$$(4xy^2 + 6y)\,dx + (5x^2y + 8x)dy = 0.$$ 6

(b) Assume that the population of a certain city increases at a rate proportional to the mumbler of inhabitants at any time. If the population doubles in 40 years, in how many years will it triple ? 6

(c) Show that e^{2x} and e^{3x} are linearly independent solutions of the equation :

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

Write the general solution that satisfies the condition $y(0) = 2$, $y'(0) = 3$. 2,2,2

5. (a) Solve :

$$x^2\frac{d^2y}{dx^2} - 5x\frac{dy}{dx} + 8y = 2x^3.$$

6½

(b) Solve :

$$[2x^2 + 2xy + 2xz^2 + 1]\ dx + dy + 2zdz = 0.$$

6½

(c) Solve the equation by the method of Variation of Parameter :

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \frac{1}{1 + e^x}.$$

6½

6. (a) (i) Form a partial differential equation by eliminating a, b, c from the relation : 2

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

(*ii*) Classify the following partial differential equation into elliptic, parabolic or hyperbolic :

(A) $x^2(y - 1)\, r - x(y^2 - 1)\, s + y\,(y - 1)t + xyp - q = 0.$ 2½

(B) $r + 2s + t = 0$ 2

where $r = \dfrac{\partial^2 z}{\partial x^2}, s = \dfrac{\partial^2 z}{\partial x \partial z}, t = \dfrac{\partial^2 z}{\partial y^2}, p = \dfrac{\partial z}{\partial x}, q = \dfrac{\partial z}{\partial y}.$

(*b*) Find the complete integral of equation :

$px + qy = pq.$ 6½

(*c*) Find the general solution of the differential equation :

$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z.$ 6½