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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **373**

Unique Paper Code : **235351** **C**

Name of the Paper : **Paper B : Integration and Differential Equations**

(Admissions of 2011 and onwards)

Name of the Course : **B.A. (Prog.) Mathematics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Find the horizontal line $y = k$ that divides the area between $y = x^2$ and $y = 9$ into two equal parts. 6

(*b*) Evaluate :

(*i*) $$\int \frac{(2x+3)\,dx}{\sqrt{3+4x-4x^2}}$$

(*ii*) $$\int_0^{\pi} \frac{x \tan x \, dx}{\sec x + \tan x}.$$ 3,3

(*c*) If

$$I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx,$$

show that :

$$n\left(I_{n-1} + I_{n+2}\right) = 1.$$

Deduce the value of I_5. 6

2. (*a*) If

$$I_{m,n} = \int \cos^m x \sin nx \, dx,$$

prove that :

$$(m+n)I_{m,n} = -\cos^m x \cos nx + mI_{m-1,n-1}$$

Hence evaluate :

$$\int_0^{\frac{\pi}{2}} \cos^5 x \sin 3x \, dx.$$ $3,3\frac{1}{2}$

(*b*) Find the volume of the solid generated by revolving the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

about *y*-axis (or minor axis). $6\frac{1}{2}$

(*c*) Prove that the length of the arc of the curve :

$$x = a\sin(2\theta)[1 + \cos(2\theta)],\ y = 2a\cos(\theta)[1 - \cos(2\theta)]$$

measure from $\theta = 0$ to $\theta = \frac{\pi}{2}$ is $\frac{4a}{3}$. $6\frac{1}{2}$

3. (*a*) Evaluate :

(*i*) $\int x^5 \sqrt{\frac{1+x^2}{1-x^2}} \, dx$

(*ii*) $\int_0^{\frac{\pi}{4}} \log(1+\tan\theta) \, d\theta.$ 3,3 (*b*) Show that the solutions $\sin 2x$, $\cos 2x$ and e^{-3x} of the equation :

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 12y = 0$$

are linearly independent and find the general solution. 3,3

(*c*) In a certain bacteria culture the rate of increase in the number of bacteria is a proportional to the number present :

(*i*) If the number triples in 5 hr, how many will be present in 10 hr.

(*ii*) When will the number present be 10 times the number initially present. 3,3

4. (*a*) Solve :

$$\left(2x^2 + y\right) dx + \left(x^2y - x\right) dy = 0.$$ 6

(*b*) Find the orthogonal trajectories of the family of curves :

$$cx^2 + y^2 = 1$$

where c is a constant. 6

(*c*) Solve :

$$y = 2px + p^4x^2, \quad p = \frac{dy}{dx}.$$ 6

5. (*a*) Solve the equation by the method of Variation of Parameter :

$$y'' + y = \tan x.$$ $6\frac{1}{2}$

(*b*) Solve :

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3.$$ $6\frac{1}{2}$

(*c*) Solve the system of equations :

$$2\frac{dx}{dt} + \frac{dy}{dt} - 3x - y = t,$$

$$\frac{dx}{dt} + \frac{dy}{dt} - 4x - y = e^t.$$ $6\frac{1}{2}$

6. (*a*) (*i*) Form a partial differential equation by eliminating *a*, *b* from the relation :

$$2z = (ax + y)^2 + b.$$ 2

(*ii*) Classify the following partial differential equation into elliptic, parabolic or hyperbolic :

(A) $y^2r - 2xys + x^2t = \frac{y^2}{x}p + \frac{x^2}{y}q$

where $r = \frac{\partial^2 z}{\partial x^2}, s = \frac{\partial^2 z}{\partial x \partial y}, t = \frac{\partial^2 z}{\partial y^2}, p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}.$ 2

(B) $\sin^2 x\frac{\partial^2 z}{\partial x^2} + 2\cos x\frac{\partial^2 z}{\partial x \partial y} - \frac{\partial^2 z}{\partial y^2} = 0.$ $2\frac{1}{2}$

(*b*) Find the general solution of the differential equation :

$$x^2p + y^2q = x + y.$$ $6\frac{1}{2}$

(*c*) Find the complete integral of :

$$2(z + xp + yq) = yp^2.$$ $6\frac{1}{2}$