[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5219 **D** **Your Roll No................**

Unique Paper Code : 235351

Name of the Course : **B.A. (Prog.)**

Name of the Paper : Integration and Differential Equations

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Show that : $\int_0^{\pi/2} \log (\tan x + \cot x)\, dx = \pi \log 2$. (6½)

(b) Find the reduction formula for $\int \cos^n x\, dx$ (n being a positive integer) and hence evaluate $\int_0^{\pi/2} \cos^n x\, dx$. (6½)

(c) Find the area included between the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ and its base. (6½)

2. (a) Find the surface area of the solid generated by revolving the curve : $x = e^t \sin t$, $y = e^t \cos t$, $0 \le t \le \pi/2$, about the axis of x. (6)

(b) Find the entire length of the astroid

$x^{2/3} + y^{2/3} = a^{2/3}$ or $x = a\cos^3\theta,\ y = a\sin^3\theta$ (6)

(c) Evaluate :

(i) $\int (2x-5)\sqrt{2+3x-x^2}\,dx$ (3)

(ii) $\int_0^{\pi/2} \frac{\sin^2 x}{1+\sin x\cos x}\,dx$ (3)

3. (a) Find the volume of the ellipsoid formed by the revolution of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, about the major axis. (6½)

(b) Solve $y'' + 4y = 4\sec^2 2x$ by the method of variation of parameters. (6½)

(c) (i) Solve : $(1 + x + xy^2)\,dy + (y + y^3)dx = 0$ (3)

(ii) Solve : $y = 2px + p^4x^2,\ p = \frac{dy}{dx}$ (3½)

4. (a) Solve $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x$. (6)

(b) Show that $e^x\sin x$, $e^x\cos x$ are linearly independent solutions of $y'' - 2y' + 2y = 0$, $\left(\text{where } y' = \frac{dy}{dx}\right)$. Find the general solution. Also find the solution $y(x)$ with the property $y(0) = 2$, $y'(0) = -3$. (6)

(c) Find the orthogonal trajectories of the family of curves :

$x^2 + y^2 = cx^3$

where c is a constant. (6)

5. (a) (i) Assume that the population of a certain city increases at a rate proportional to the number of inhabitants at any time. If the population doubles in 40 years, in how many years will it triple itself. (2)

(ii) Find the Particular Integral $\frac{d^2y}{dx^2} + y = \cos^2 x$. (4½)

(b) Solve the differential equation.

$\frac{dx}{dt} - y = t^2, \frac{dy}{dt} + 4x = t$ (6½)

(c) Verity the condition of integrability for the differential equation

$(2x + y^2 + 2xz)\, dx + 2xy\, dy + x^2dz = 0$, also solve it. (6½)

6. (a) (i) Form a partial differential equation by eliminating a, b from the equation $z = (x + a)(y + b)$.

(ii) Classify the following partial differential equation into Elliptic, Parabolic, Hyperbolic

$x^2(y - 1)\, r - x(y^2 - 1)\, s + y(y - 1)\, t + xyp - q = 0$

where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$, $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$ (3,3)

(b) Find the general solution of the differential equation

$x^2(y - z)\,p + y^2 + z - x)\,q = z^2(x - y)a$ (6)

(c) Find the complete integral of

$2zx - px^2 - 2pxy + pq = 0$ (6)