[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5220 **D** **Your Roll No...............**

Unique Paper Code : 235351

Name of the Course : **B.A. (Prog.)**

Name of the Paper : Integration and Differential Equations

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) If $U_n = \int_0^{\pi/2} \theta \sin^n \theta \, d\theta \, (n > 1)$, then

   $U_n = \frac{n-1}{n} U_{n-2} + \frac{1}{n^2}$. Deduce that $U_5 = \frac{149}{225}$. (6½)

   (b) Show that : $\int_0^{\pi/2} \log \sin x \, dx = \frac{\pi}{2} \log \frac{1}{2}$. (6½)

   (c) Find the surface area of the solid generated by revolving the curve :

   $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$ about x-axis. (6½)

2. (a) Prove that the length of the arc of the curve $x = a \sin 2\theta \, (1 + \cos 2\theta)$, $y = a \cos 2\theta \, (1 - \cos 2\theta)$ measured from $\theta = 0$ to $\theta = \frac{\pi}{2}$ is $\frac{4a}{3}$.

   (6½)

(b) The region enclosed by the curves y = sin x, y = cos x and the x-axis between $x = 0$ to $x = \frac{\pi}{2}$, is revolved about the x-axis. Find the volume of the solid thus generated. (6½)

(c) Find the area of the smaller portion enclosed by the curves :

$x^2 + y^2 = 9,\ y^2 = 8x.$ (6½)

3. (a) Evaluate :

(i) $\int_0^{\pi/2} \log \tan x$

(ii) $\int_1^2 \frac{dx}{(x+1)\sqrt{x^2-1}}$ (3+3)

(b) Solve : $\frac{d^2y}{dx^2} + y = \tan x$ using the method of variation of parameters. (6)

(c) (i) Solve : $(xy^3 + y)\,dx + (2x^2y^2 + x + y^4)\,dy = 0$

(ii) Solve : $y = 2px - xp2$, where $p = \frac{dy}{dx}$. (3+3)

4. (a) Solve : $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 24x^2$.

(b) Show that sin x, cos x and sin x – cos x are solution of the differential equation $y'' + y = 0$ where $y' = dy/dx$. Are these solutions linearly dependent ? (Use the idea of Wronskian) (6)

(c) Find the orthogonal trajectories of the family of curve.

$x^2 + y^2 = 2ax$ (6)

5. (a) (i) A culture initially has No number of bacteria. At t = 1 hour, the number of bacteria is measured to be $\frac{3}{2}$ No. If the rate of growth is proportional to the number of bacteria present. Determine the time necessary for the number of bacteria of to triple.

(ii) Find the particular integral $\frac{d^3y}{dx^3} + y = x$. (4,2½)

(b) Solve the simultaneous equations :

$$\frac{dx}{dt} + \frac{dy}{dt} - 2y = 2\cos t - 7\sin t$$

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x = 4\cos t - 3\sin t$$ (6½)

(c) Verity the condition of inerrability for the differential equation $yz \log zdx - zx \log z\, dy + xydz = 0$, also solve it. (6½)

6. (a) (i) Form a partial differential equation by eliminating arbitrary function

$z = xy + f(x^2 + y^2)$

(ii) Classify the following partial differential equation into elliptic, parabolic, hyperbole

$$x(xy - 1)r - (x^2y^2 - 1)s + y(xy - 1)t + px + yq = 0$$

where $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

OR

$$\sec^2 \frac{\partial^2 z}{\partial x^2} + 2\tan x \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$$ (3,3)

(b) Find the general solution of the differential equation

$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$$ (6)

(c) Find the complete integral of

$$x^2p^2 + y^2q^2 = z^2$$ (6)