This question paper contains 7 printed pages]

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Six* questions in all.

Q. No. **1** is compulsory.

Simple calculator can be used.

Attempt *Five* more questions.

1. (*a*)   Fill in the blanks :

   (*i*)   *t*-distribution reduces to Cauchy distribution for......................d.f.

   (*ii*)   If χ_1^2 and χ_2^2 are two independent Chi-square variates with n and m d.f., respectively, then $\dfrac{\chi_1^2}{\left(\chi_1^2 + \chi_2^2\right)} \sim$ .....

   (*iii*)   Yule's coefficients of association Q = ...

P.T.O.

(b) Examine the consistency of the following data :

N = 1000; (A) = 600, (B) = 500, (AB) = 50, the symbols having their usual meaning.

(c) If X ~ F(n, m), then show that :

$$\left(\frac{1}{X}\right) \sim F(m, n).$$

(d) Let X, Y, Z, W be i.i.d. $N(0, \sigma^2)$ variates and $T = \dfrac{k(X + Y)}{\sqrt{Z^2 + W^2}}$. For what value of k, $T \sim t_{(n)}$. Find n as well.

(e) Find the Mode of $X \sim \chi^2_{(n)}$.   3×5=15

2. (a) Investigate the association between darkness of eye-colour in father and son from the following data :

Father with dark eyes and sons with dark eyes = 50

Father with dark eyes and sons with not dark eyes = 79

Father with not dark eyes and sons with dark eyes = 89

Father with not dark eyes and sons with not dark eyes = 782

What would have been the frequency of *fathers with dark eyes and sons with dark eyes*' for the same total number, had there been complete independence ?

(b) Show that :

$$(AB)^2 + (\alpha\beta)^2 - (\alpha B)^2 - (A\beta)^2 = [(A) - (\alpha)][(B) - (\beta)] + 2N\delta,$$

the symbols having their usual meaning.   6,6

3. (a) If $X \sim \chi^2_{(n)}$. Obtain m.g.f. of X. Hence m.g.f. of standard chi-square variate and obtain its limiting form as $n \to \infty$. Also interpret the result.

(b) In a locality, 100 persons were randomly selected and asked about their education achievements. The results were given below :

Education

Sex	Middle	High School	College
Male	10	15	25
Female	25	10	15

Compute the statistic you would use to test whether the education pattern varies with sex. Also state how you would proceed further.   6,6

4. (a) Define sampling distribution and standard error. Obtain the expressions for unbiased estimates of population mean and population variance. Assume that n, the sample size, is large.

(b) For the t-distribution with n degrees of freedom establish the recurrence relations :

$$\mu_{2r} = \frac{n(2r - 1)}{(n - 2r)} \cdot \mu_{2r-2}, \; n > 2r.$$ 6,6

5. (a) For a chi-square distribution with n degrees of freedom establish the following recurrence relations between the moments : $\mu_{r+1} = 2r(\mu_r + n\mu_{r-1}), \; r \geq 1$. Hence find β_1 and β_2.

(b) Below are given the gain in weights in (lbs) of pigs fed on two diets :

Diet A	Diet B
25	44
32	34
30	22
34	10
24	47
14	31
32	40
24	30

30	32
31	35
35	18
25	21
	35
	29
	22

Test if the two diets differ significantly as regards their effect on increase in weight at 0.05 level of significance.   6,6

6.   (a)   Show that for a t-distribution with n degrees of freedom, mean deviation about mean is given by :

$$\sqrt{(n)}\,\Gamma[(n-1)/2] / \sqrt{\pi}\,\Gamma(n/2).$$

(b)   If $X \sim F(n, m)$, prove that for $n = m$, the quartiles Q_1, Q_2 and Q_3 satisfies the condition $Q_1 Q_3 = Q_2$.   6,6

7. (*a*) The following random samples are measurements of the heat-producing capacity (in millions of calories per ton) of specimens of coal from two mines :

Mine 1	Mine 2
8,260	7,950
8,130	7,890
8,350	7,900
8,070	8,140
8,340	7,920
	7,840

Test whether it is reasonable to assume that the variances of the two populations sampled are equal.

(*b*) In a year there are 956 births in a town A, of which 52.5% were males, while in towns A and B combined, this proportion in a total of 1,406 births was 0.496. Is there any significance difference in the proportion of male birth in the two towns ? 6,6

8. Write short notes on :

    (*a*)   Yates correction for continuity;

    (*b*)   Paired *t*-test for difference of means;

    (*c*)   Relation between F and χ^2 distribution.   4,4,4