[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 102 **E** **Your Roll No...............**

Unique Paper Code : 235351

Name of the Course : **B.A. (Prog.)**

Name of the Paper : Paper B : Mathematics
Integration and Differential Equations

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any two parts from each question.

1. (a) Show that the length the of the curve $y = \log\left(\frac{e^x - 1}{e^x + 1}\right)$ from $x = 1$ to $x = 2$ is $\log(e + e^{-1})$. (6½)

   (b) Find the area included between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (6½)

   (c) Find the volume of the solid obtained by revolving one arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ (6½)

2. (a) Obtain a reduction formula for $\int e^{ax} \sin^n x \, dx$, n being a positive integer. (6)

P.T.O.

(b) Show that : $\int_0^{\pi/2} \log \cos x \, dx = \frac{\pi}{2} \log \frac{1}{2}$. (6)

(c) Evaluate :

(i) $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ (3)

(ii) $\int \frac{(x+2)dx}{\sqrt{4+3x-2x^2}}$ (3)

3. (a) Find the surface of the solid generated by the revolution of the astroid : $x = a\cos^3 t$, $y = a\sin^3 t$ about x-axis. (6½)

(b) Solve the differential equation :

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x + \sin x$$

by the method of variation of parameter (6½)

(c) (i) Solve : $(3x^3 + 4xy)\,dx + (2x^2 + 2y)\,dy = 0$ (3)

(ii) Solve : $p^3y^2 - 2px + y = 0,\ p = \frac{dy}{dx}$ (3½)

4. (a) Solve :

$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x)$$ (6)

(b) Using the concept of Wronskian, show that e^{2x} and e^{3x} are linearly independent solutions of the differential equation $y'' - 5y' + 6y = 0$ where $y' = dy/dx$. Find the general solution $y(x)$ satistbying the conditions $y(0) = 0$ and $y'(0) = 1$. (6)

(c) Find the orthogonal trajectories of the parabola $6ay^2 = (x - 3)$ where a is variable parameter. (6)

5. (a) (i) Bacteria in a certain culture increase at a rate proportional to the number present. If the number N increase from 1000 to 2000 in 1 Hour. How many are present at the end of 1.5 hours ?

(ii) Find Particular Integral $\frac{d^2y}{dx^2} + y = \sin^2 x$. (4½)

(b) Solve the differential equations

$$\frac{dx}{dt} + \frac{dy}{dt} + 3x = \sin t$$

$$\frac{dx}{dt} + y - x = \cos t$$ (6½)

(c) Verify the condition of integrability for the differential equation $(x - y)dx - xdy + zdz = 0$. Also solve it. (6½)

6. (a) (i) From a partial differential equation by eliminating a, b from the equation :

$$(x + a)^2 + (y + b)^2 + z^2 = 1$$

(ii) Classify the following partial differential equation into elliptic, parabolic, hyperbolic :

$$xyr - (x^2 - y^2)s - xyt + py - qx = 2(x^2 - y^2)$$

where $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

(b) Find the general solution of the differential equation

$x(y - z)p + y(z - x)q = z(x - y)$. (6½)

(c) Find the complete integral of :

$p^2z^2 + q^2 = 1$ (6½)