[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 103 E **Your Roll No................**

Unique Paper Code : 235351

Name of the Course : **B.A. (Prog.)**

Name of the Paper : Paper D : Mathematics
Integration and Differential Equations

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) If $I_{m,n} = \int \cos^m x \sin nx \, dx$, prove that

$$(m + n)\, I_{m,n} = -\cos^m x \cos nx + m\, I_{m-1,\, n-1}$$

Hence evaluate $\int_0^{\pi/2} \cos^5 x \sin 3x \, dx$. (3½+3)

(b) If $I_n = \int_0^{\pi/4} \tan^n x \, dx$, show that for $n > 1$, $I_n + I_{n-2} = \frac{1}{n-1}$. Deduce the value of I_5. (3½+3)

(c) Find the area of the smaller portion enclosed by the curves :

$x^2 + y^2 = 16;\ y^2 = 6x$ (6½)

2. (a) Find the volume of the solid generated by revolving the ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, about the y-axis (or minor axis). (6½)

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(b) Find the surface of the solid generated by the revolution of the astroid :

$x = a \cos^3\theta$, $y = a \sin^3\theta$, about x-axis. (6½)

(c) Find the length of the are of the parabola $y^2 = 4ax$ cut off by its latus rectum. (6½)

3. (a) Evaluate :

(i) $$\int \frac{2x+3}{\sqrt{3+4x-4x^2}}\,dx$$

(ii) $$\int_0^\pi \frac{x\,dx}{a^2\cos^2 x + 5^2\sin^2 x}$$ (3+3)

(b) (i) Solve : $(4xy^2 + 6y)dx + (5x^2y + 8x)dx$

(ii) Solve : $y = 2px + p^4x^2,\ p = \frac{dy}{dx}$ (3+3)

(c) Solve the equation :

$$\frac{d^2y}{dx^2} + 4y = \sin^2 2x$$

by the method of variation of parameters. (6)

4. (a) Solve $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\log x}{x^2}$. (6)

(b) Evaluate Wronskian of the function $y_1(x) = \sin x$ and $y_2(x) = \sin x - \cos x$ and henes conclude whether or not they are linearly independent. Also form the differential equation. (6)

(c) Find the orthogonal trajectories of the family of curves $3xy = x^3 - a^3$, a being parameter of the family. (6)

5. (a) (i) In a culture of yeast, the amount A of active yeast grows at a rate proportional to the amount present. If the original amount A_0 double in 2 hours. How long does it take for the original amount to triple ?

(ii) Find the particular integral $\frac{d^3y}{dx^3} - y = 1$. (4,2½)

(b) Solve the system of equation

$$\frac{dx}{dt} + 4x + 3y = t$$

$$\frac{dy}{dt} + 2x + 5y = e^t$$

(6½)

(c) Verify the condition of inerrability for the differential equation

$2yz\,dx + zxdy = xy(1 + z)dz = 0$ (6½)

6. (a) (i) Form a partial differential equation by eliminating a, b, c from the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

(ii) Classify the following partial differential equation into elliptic, parabolic, hyperbolic

$$x^2r + \frac{5}{2}xys + y^2t + xp + yq = 0$$

where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}, r = \frac{\partial^2 z}{\partial x^2}, s = \frac{\partial z}{\partial xy}, t = \frac{\partial^2 z}{\partial y^2}$ (4,2½)

(b) Find the general solution of the differential equation

$(y^2 + z^2 - x^2)p - 2xyq + 2xz = 0$ (6½)

(c) Find the complete integral of

$pxy + pq + qy - yz = 0$ (6½)