[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1706 **F-3** **Your Roll No...............**

Unique Paper Code : 2352401

Name of the Course : **B.A./B.Sc. (H) – Allied Courses**
[For the Students of other than Mathematics]

Name of the Paper : Linear Algebra

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting two parts each from each section.

1. (a) Explain the method of "Proof by Contrapositive". Use this method to prove that if x is a vector such that $||x|| = 0$, then $x = 0$. (6)

   (b) Define the projection vector of a vector b onto a vector a. Calculate $\text{proj}_a b$ and verify that the vector $b - \text{proj}_a b$ is orthogonal to a where $a = (2,1,5)$ and $b = (1, 4, -3)$. (6)

   (c) Find the rank of the matrix A given by $\begin{pmatrix} 1 & -1 & 3 & 4 \\ 2 & 0 & 4 & -3 \\ -1 & -3 & 1 & 3 \end{pmatrix}$. (6)

2. (a) Determine whether $(5, 17, -20)$ is in the row space of the following matrix

$$A = \begin{pmatrix} 3 & 1 & -2 \\ 4 & 0 & 1 \\ -2 & 4 & -3 \end{pmatrix}$$ (6½)

   (b) Prove that $\mathbb{R}^3$ is a vector space with respect to the usual operations of vector addition and scalar multiplication in $\mathbb{R}^3$. (6½)

(c) Prove that the subset $W = \left\{\left(a, b, \frac{1}{2}a - 2b\right); a, b \in \mathbb{R}\right\}$ of $\mathbb{R}^3$ is a subspace of $\mathbb{R}^3$. (6½)

3. (a) Show that the set $S = \{x^2 + x + 1, x + 1, 1\}$ spans $\mathcal{P}_2$, where $\mathcal{P}_2$ is the vector space of all polynomials of degree at most 2. (6)

(b) Show that the set $B = \{(1,2,1), (2,3,1), (-1,2,-3)\}$ is a basis for $\mathbb{R}^3$. (6)

(c) Check the consistency of the following system using rank criterion. Hence solve the system, if consistent.

$$\begin{pmatrix} 2 & 1 & 3 \\ 1 & -2 & 2 \\ 0 & 1 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$ (6)

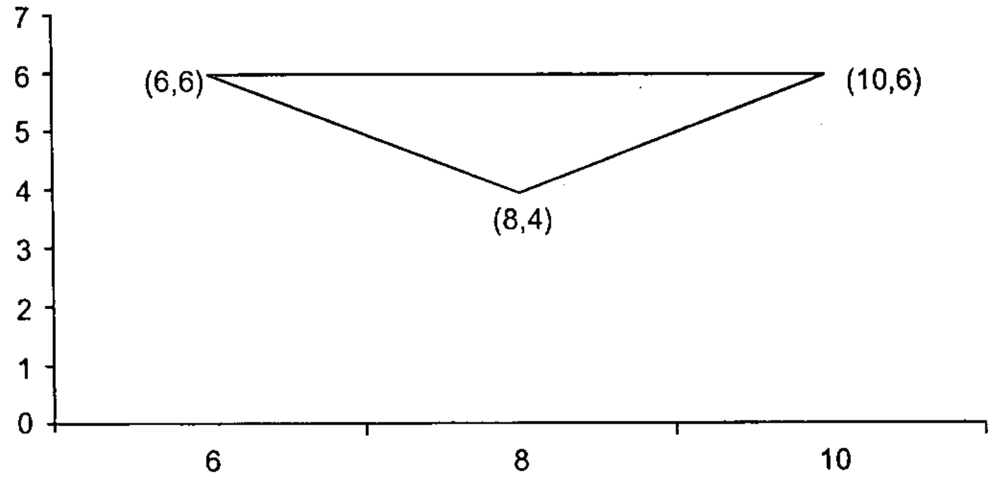
4. (a) Let $S = \{v_1, v_2, v_3, v_4\}$ be a basis of $\mathbb{R}^4$, where $v_1 = (1,1,0,0)$, $v_2 = (2,0,1,0)$, $v_3 = (0,1,2,-1)$, $v_4 = (0,1,-1,0)$. If $v = (1,2,-6,2)$, compute the coordinate vector $[v]_S$ of v with respect to the ordered basis S. (6½)

(b) Show that the function $g: \mathcal{P}_n \to \mathcal{P}_{n-1}$ given by $g(p) = p'$, $p \in \mathcal{P}_n$, is a linear transformation where $\mathcal{P}_n$ is the vector space of all polynomials of degree at most n and p' is the derivative of p. (6½)

(c) Let $L: \mathbb{R}^3 \to \mathbb{R}^2$ defined as $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be a linear transformation and $S = \{v_1, v_2, v_3\}$ and $T = \{w_1, w_2\}$ be bases of $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively, where $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $w_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $w_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

Find the matrix of L with respect to S and T. (6½)

5. (a) For the graphic in the following figure, use ordinary coordinates in $\mathbb{R}^2$ to find new vertices after performing each indicated operation. Then sketch the figures that would result from each movement.



(i) translation along the vector (–3,5).

(ii) scaling about the origin with scale factors of $\frac{1}{2}$ in the x-direction and 3 in the y-direction. (6½)

(b) Find the kernel and range of the linear transformation $L : \mathbb{R}^2 \to \mathbb{R}^3$ given by $L(x, y) = (x, x + y, y)$. Also find the dimension of kernel and range of L. (6½)

(c) State the dimension theorem. Hence or otherwise, determine the nullity of the linear transformation $L : \mathbb{R}^3 \to \mathbb{R}^2$ defined as $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 3 & 2 & 11 \\ 2 & 1 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$. (6½)

6. (a) Check whether the following linear transformation $L : \mathbb{R}^3 \to \mathbb{R}^3$ is one to one or not.

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -7 & 4 & -2 \\ 16 & -7 & 2 \\ 4 & -3 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$ (6½)

(b) Show that the set $S = \{(1,0,-1), (-1,4,-1), (2,1,2)\}$ is an orthogonal basis of $\mathbb{R}^3$. Hence find the orthonormal basis of $\mathbb{R}^3$. (6)

(c) Find the orthogonal component W^1 of the subspace W of $\mathbb{R}^3$ spanned by the set $\{(1,4,-2), (2,1,-1)\}$. (6)