[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 106 **E** **Your Roll No.....**

Unique Paper Code : 236351

Name of the Course : **B.A. Programme – Operational Research**

Name of the Paper : Mathematical Programming

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** questions.
3. Use of non-programmable calculator is allowed.

1. (a) What is an unconstrained optimization problem ? Define the concept of stationary point and relative extrema. (7)

   (b) Determine the relative maximum and minimum (if any) of the following function :

   $$f(x_1, x_2, x_3) = x_1 + 2x_2 + x_2x_3 - x_1^2 - x_2^2 - x_3^2$$ (8)

2. (a) Define a convex function and prove that a positive linear combination of convex functions is again a convex function. (6)

   (b) Use the Kuhn-Tucker conditions to solve the following non-linear programming problem :

P.T.O.

Maximize $Z = 2x_1 - x_1^2 + x_2$

Subject to constraints

$$2x_1 + 3x_2 \leq 6$$

$$2x_1 + x_2 \leq 4$$

$$x_1 \geq 0, \; x_2 \geq 0$$ (9)

3. Use the Wolfe's method for solving the following quadratic problem

Maximize $Z = 2x_1 + 3x_2 - 2x_1^2$

Subject to $x_1 + 4x_2 \leq 4$

$$x_1 + x_2 \leq 2$$

$$x_1, x_2 \geq 0$$ (15)

4. (a) What is an Integer Programming Problem ? Give some applications of integer programming problems. (7)

(b) Solve the following non-linear programming problem using the method of Lagrangian multipliers.

Maximize $Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

subject to $x_1 + 2x_2 = 2$, and $x_1, x_2 \geq 0$. (8)

5. Use the branch and bound technique to solve the following Integer programming problem

Maximize $Z = x_1 + x_2$

Subject to $2x_1 + 5x_2 \leq 16$

$6x_1 + 5x_2 \leq 30$

$x_1, x_2 \geq 0$, and are integers.

Given non-integer optimum solution is : $x_1 = 7/2$, $x_2 = 9/5$ and Maximum $Z = 53/10$. (15)

6. (a) The following table provides a non- integer Optimum solution to IPP :

Maximize $Z = 4x_1 + 2x_2$

Subject to

$x_1 + x_2 \leq 7$

$2x_1 \leq 11$

$2x_2 \leq 7$

$x_1, x_2 \geq 0$, and are integers.

Optimum feasible non-integer solution

	c_j	4	2	0	0	0	
C_B	X_B	x_1	x_2	s_1	s_2	s_3	b
1	x_1	1	0	1	0	–0.5	3.5
0	s_2	0	0	–2	1	1	4
2	x_2	0	1	0	0	0.5	3.5

Find the integer solution by using Gomory's fractional cut algorithm for mixed integer programming problem. (8)

(b) Prove that the following function is convex :

$$f(x_1, x_2) = 2x_1^2 + 3x_1x_2 + 2x_2^2 - 10x_1 - 10x_2, (x_1, x_2) \in R^2.$$ (7)

7. Write a short note on :–

(a) Branch and Bound Algorithm

(b) Bordered Hessian matrix

(c) Convex and Concave functions (15)