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Sr. No. of Question Paper : 5178 F Your Roll No.....

Unique Paper Code : 235351

Name of the Paper : Paper B : MT Integration and Differential Equations

Name of the Course : B.A. (Prog.) Mathematics

Semester : III

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any two parts from each question.

1. (a) Find the area of the region enclosed between the curves $y = x^2$ and $y = x + 6$. (6)

(b) Find the volume of the solid that results when the region enclosed by the given curve is revolved about y-axis

$$y = x^2 - 1, x = 2, y = 0 \quad (6)$$

(c) Find the circumference of a circle of radius a from the parametric equations $x = a \cos t, y = a \sin t$ ($0 \leq t \leq 2\pi$) (6)

2. (a) Evaluate :

$$\int \frac{x}{(x-3)\sqrt{x+1}} dx \quad (6\frac{1}{2})$$

(b) Find the reduction formula for

$$I_n = \int \cos^n x \text{ and hence evaluate}$$

$$I_5 = \int \cos^5 x dx \quad (6\frac{1}{2})$$

P.T.O.

(c) Find the reduction formula for

$$I_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x \, dx \quad (6\frac{1}{2})$$

3. (a) Solve :

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0,$$

$$\frac{dy}{dt} + 5x + 3y = 0. \quad (6)$$

(b) Find the arc length of the curve $y = x^{3/2}$ from $(1,1)$ to $(2, 2\sqrt{2})$ using formula

$$\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx \quad (6)$$

(c) Evaluate the integration

$$I = \int \frac{(2x+1)}{(2x^2+3x+2)^3} \, dx \quad (6)$$

4. (a) Solve any one of the following :

(i) $(x+2y-1)dx = (x+2y+1)dy$

(ii) $(y^4+2y)dx + (xy^3+2y^4-4x)dy = 0$

(iii) $p^3 - 4xy p + 8y^2 = 0 \quad (6\frac{1}{2})$

(b) Solve :

$$(x^2D^2 - 3xD + 5)y = x^2 \sin(\log x)$$

OR

Solve :

$$(D^2 - 1)y = x^2 \cos x \quad (6\frac{1}{2})$$

- (c) Using the concept of Wronskian, show that e^{2x} and e^{3x} are linearly independent solutions of the differential equation

$$y'' - 5y' + 6y = 0, \text{ where } y' = \frac{dy}{dx}$$

Find the general solution $y(x)$ satisfying the conditions $y(0) = 0$ and $y'(0) = 1$. (6 $\frac{1}{2}$)

5. (a) Solve :

$$\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$$

OR

In a certain bacteria culture, the rate of increase in the number of bacteria is proportional to the number present. If the number triples in 5 hours, how many will be present in 10 hours ? (6)

- (b) Verify the condition of integrability and then solve the following differential equation :

$$zy (1 + 4xz) dx - xz (1 + 2xz) dy - xy dz = 0. \quad (6)$$

- (c) Apply the method of variation of parameter to

$$y'' + 4y = 4 \tan 2x. \quad (6)$$

6. (a) Form a partial differential equation by eliminating a, b, c from the equations

(i) $z = ax^2 + by^2 + ab$

(ii) $z = ax + by + cxy$ (6 $\frac{1}{2}$)

P.T.O.

(b) Find the Lagrange's method the general solution of the equation

$$2p + 3q = 1 \quad (6\frac{1}{2})$$

(c) Solve the equation by Charpit's method

$$q = -xp + p^2 \quad (6\frac{1}{2})$$