[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5177 **F** **Your Roll No...............**

Unique Paper Code : 235351

Name of the Paper : MT–Paper B : Integration and Differential Equations

Name of the Course : **B.A. (Prog.) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.

1\. (a) Find the area of the region enclosed by the parabola $y = 2x - x^2$ and the x-axis. (6)

(b) Find the volume of the solid that results when the region enclosed by the given curve is revolved about x-axis

$$y = \sqrt{25 - x^2},\ y = 3. \qquad (6)$$

(c) Find the area of the surface generated by revolving the given curve about the x-axis

$$y = \sqrt{x},\ 1 \le x \le 4. \qquad (6)$$

2\. (a) Evaluate : $\int \frac{x\,dx}{(x+2)\sqrt{(x+1)}}$ (6½)

(b) If $I_n = \int_0^{\Pi/4} \tan^n \theta d\theta$, show that

$$n(I_{n-1} + I_{n+1}) = 1$$ (6½)

(c) Evaluate $\int_0^{2a} x^2 \sqrt{2ax - x^2}\, dx$. (6½)

3. (a) Solve :

$$\frac{dx}{dt} + 4y = \sec^2 2t,$$

$$\frac{dy}{dt} = x.$$ (6)

(b) Verify the condition of integrability for the differential equation

$$(x - y)dx - x\,dy + z\,dz = 0.$$

Also find its solution. (6)

(c) Integrate the following :

$$I = \int_0^{\pi/2} \sin^3 x \cos^4 x dx.$$ (6)

4. (a) Define ordinary differential equation, its order and degree.

OR

Solve :

$$\left(x^4 - 2xy^2 + y^4\right)dx = \left(2x^2y - 4xy^3 + \sin y\right)dy.$$ (6½)

(b) Solve any one of the following : (6½)

(i) $3e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0$

(ii) $y = 2px + y^2p^3$

(c) Solve : (6½)

$(x^2 + y^2 + x)\, dx + xy\, dy = 0.$

OR

Find the orthogonal trajectories of the family of curves :

$$cx^2 + y^2 = 1.$$

5. (a) Solve :

$(D^3 - 7D - 6)y = e^{2x}(1 + x).$

OR

Solve :

$$x\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = \frac{1}{x}.$$ (6)

(b) Apply the method of variation of parameters to solve

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2e^x.$$ (6)

(c) State a necessary and sufficient condition for the solutions $y_1(x)$ and $y_2(x)$ of equation

$a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$ to be linear independent. Show that e^{2x} and e^{3x} are linearly independent solution of equation $y'' - 5y' + 6y = 0$ on $-\infty < x < \infty$. What is its general solution ? Find the solution y(x) that satisfies conditions $y(0) = 2$, $y'(0) = 3$. (6)

6. (a) Form partial differential equations from the following equations by eliminating the arbitrary constants

(i) $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

(ii) $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (6½)

(b) Solve by Lagrange's method the following equations

(i) $\frac{y^2z}{x}p + xzq = y^2$

(ii) $(x^2-yz)\ p + (y^2-zx)\ q = z^2-xy.$ (3,3½)

(c) Solve by Charpit's method the following equation :

$\left(p^2+q^2\right)y = qz.$ (6½)