[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5181 **F** **Your Roll No...............**

Unique Paper Code : 236351

Name of the Paper : OPR : Operational Research – Mathematical Programming

Name of the Course : **B.A. Programme**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** questions.
3. Use of non-programmable calculator is allowed.

1. (a) Differentiate between constrained and unconstrained optimization problem. Give the necessary and sufficient conditions for a stationary point to be an extremum of an unconstrained problem. (7)

   (b) The production function of a commodity is given by : $Q = 40F + 3F^2 + (F^3/3)$, where Q is the total output and F is the unit of inputs. Find the number of units of input required to give the maximum output. (8)

2. (a) Let f(x) be defined and differentiable on an open convex set S. Show that f(x) is convex if

$$f(x_2) - f(x_1) \geq (x_2 - x_1)^T \ \nabla f(x)$$

for all $x_1, x_2 \in S$. (7)

P.T.O.

(b) Solve the following non-linear programming problem, using the method of Lagrangian multipliers.

Minimize $Z = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200$

subject to $x_1 + x_2 + x_3 = 11$, and $x_1, x_2, x_3 \geq 0$. (8)

3. Define a Quadratic programming problem. Solve the following quadratic programming problem using Wolfe's method :

Maximize $Z = 15x_1 + 30x_2 + 4x_1x_2 - 2x_1^2 - 4x_2^2$

Subject to $x_1 + 2x_2 \leq 30$

$x_1, x_2 \geq 0$ (15)

4. (a) Use the Kuhn-Tucker conditions to solve the following non-linear programming problem :

Maximize $Z = 2x_1 - x_1^2 + x_2$

Subject to constraints

$2x_1 + 3x_2 \leq 6$

$2x_1 + x_2 \leq 4$

$x_1 \geq 0,\ x_2 \geq 0$ (9)

(b) Prove that the following function is convex :

$f(x_1, x_2) = x_1^2 + 2x_1x_2 + 2x_2^2 - 5x_1 + 4x_2,\ (x_1, x_2) \in R^2.$ (6)

5. (a) Describe an Integer programming problem and differentiate between All integer and Mixed integer programming problem. (7)

(b) The following table provides a non- integer Optimum solution to IPP :

Maximize $Z = 4x_1 + 6x_2 + 2x_3$

Subject to $4x_1 - 4x_2 \leq 5$

$-x_1 + 6x_2 \leq 5$

$-x_1 + x_2 + x_3 \leq 0$

$x_1, x_2, x_3 \geq 0$, x_1, x_2 integer

Optimum feasible non- integer solution

	Cj	4	6	2	0	0	0	
C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	b
4	x_1	1	0	0	3/10	1/5	0	5/2
6	x_2	0	1	0	1/20	1/5	0	5/4
2	x_3	0	0	1	1/4	0	1	25/4

Find the integer solution by using Gomory's fractional cut algorithm for mixed integer programming problem. (8)

6. (a) Use the branch and bound technique to solve the following problem

Maximize $Z = x_1 + x_2$

Subject to $3x_1 + 2x_2 \leq 12$

$x_2 \leq 2$

$x_1 \geq 0$, $x_2 \geq 0$ and are integers

Given non-integer optimal solution is $x_1 = 1/3$, $x_2 = 2$ and Max $Z = 5/3$. (10)

(b) Explain zero-one programming problem and utilize it to formulate the fixed charge problem. (5)

7. Write a short note on :

(a) Convex and concave functions

(b) Positive semi definite matrix and quadratic forms

(c) Branch and Bound Algorithm (15)