*This question paper contains 3 printed pages.* *Your Roll No. .....*

*Sl. No. of Ques. Paper* : **2320** **GC-3**
*Unique Paper Code* : **62354343**
*Name of Paper* : **Analytical Geometry and Applied Algebra**
*Name of Course* : **B.A. (Prog.) Mathematics (CBCS)**
*Semester* : **III**
*Duration* : : **3 hours**
*Maximum Marks* : **75**

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

*All questions are compulsory.*
*Attempt any two parts from each question.*

**SET-C**

1. (a) Identify and sketch the curve:
$$x = y^2 - 4y + 2$$
and also label the focus, vertex and directrix. 6

(b) Describe the graph of the curve:
$$3(x+2)^2 + 4(y+1)^2 = 12$$
Also find its centre and foci. 6

(c) Describe the graph of the hyperbola:
$$x^2 - y^2 - 4x + 8y - 21 = 0$$
And sketch its graph. 6

2. (a) Find the equation of the parabola that has its vertex at $(1,2)$ and focus at $(4,2)$. Also state the reflection property of parabola. 6

(b) Find the equation of the ellipse whose length of major axis is 26 and foci $(\pm 5, 0)$ and also sketch it. 6

(c) Find and sketch the curve of the hyperbola whose foci are $(6,4)$ and $(-4,4)$ and eccentricity is 2. 6

3. (a) Consider the equation:
$$3x^2 + 2xy + 3y^2 = 19.$$

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Rotate the coordinate axes to remove the xy-term. Then identify the type of conic represented by the equation and sketch its graph. 6

(b) Let an x'y' - coordinate system be obtained by rotating an xy - coordinate system through an angle $\theta = 30°$.

(i) Find the x'y' - coordinate of the point whose xy - coordinates are (2, 4).
(ii) Find an equation of the curve $2x^2 + 2\sqrt{3}xy = 3$ in x'y' - coordinates . 6

(c) Find the equation of two spheres that are centered at the origin and are tangent to the sphere of radius 1 centered at $(0,0,7)$. 6

4(a) (i) Find a vector of length 9 and oppositely directed to $\mathbf{v} = -5\mathbf{i} + 4\mathbf{i} + 8\mathbf{k}$.

(ii) Sketch the surface $2x + z = 3$ in 3-space. $3+3\frac{1}{2}$

(b) (i) Find the vector component of $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ orthogonal to $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 28\mathbf{k}$.

(ii) Find the area of triangle with vertices P( 2, 0, −3), Q( 1,4, 5), R( 7, 2, 9). $3+3\frac{1}{2}$

(c) Prove that

$$\|\mathbf{u}+\mathbf{v}\|^2 + \|\mathbf{u}-\mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2$$

and interpret the result geometrically. $6\frac{1}{2}$

5 (a) Let $L_1$ and $L_2$ be the lines whose parametric equations are

$L_1$ : $x = 4t \quad y = 1-2t \quad z = 2 + 2t$

$L_2$ : $x = 1 + t \quad y = 1 - t \quad z = -1 + 4t$

(i) Show that the lines $L_1$ and $L_2$ intersect at the point (2, 0, 3).
(ii) Find the parametric equation of line that is perpendicular to $L_1$ and $L_2$ and passes through their point of intersection. $3+3\frac{1}{2}$

(b) (i) Determine whether the points $P_1$ ( 6, 9, 7) , $P_2$ (9, 2, 0) and $P_3$ (0, −5, −3) lie on the same line.

(ii) Where does the line

$x = 2 - t, \quad y = 3t, \quad z = -1 + 2t$

intersect the plane $2y + 3z = 6$. $3+3\frac{1}{2}$

(c) (i) Find the equation of the plane through (1, 4, 3) that is perpendicular to the line

$x = 2 + t, \quad y + 3 = 2t, \quad z = -t.$

(ii) Determine whether the planes

$3x - 2y + z = 1$ , $4x + y - 2z = 4$

are parallel, perpendicular or neither. $3+3\frac{1}{2}$

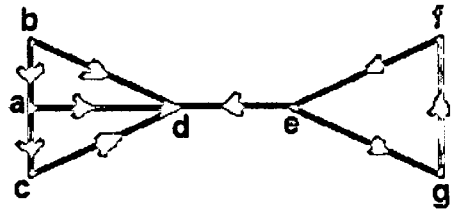
6. (a) Given three containers 3, 7, and 10 liters respectively with the largest being full of water, determine a minimum sequence of pouring method of dividing this quantity of water into two equal amounts of 5 liters using the three containers and no other measuring devices. $6\frac{1}{2}$

(b) Is the following square a Latin square? Can it be a group with the multiplication operation defined?

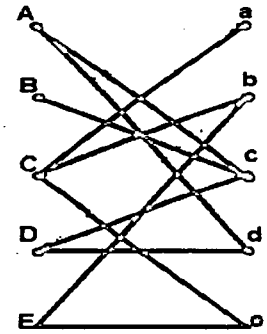
| * | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 |
| 2 | 2 | 1 | 4 | 5 | 3 |
| 3 | 3 | 4 | 5 | 2 | 1 |
| 4 | 4 | 5 | 1 | 3 | 2 |
| 5 | 5 | 3 | 2 | 1 | 4 |

$6\frac{1}{2}$

(c) (i) Given the influence model. Find the sets of minimum number of vertices which can influence every other vertex in the graph.




(ii) Find a matching or explain why none exists for the following graph.




$3+3\frac{1}{2}$

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