

This question paper contains 4 printed pages]

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S. No. of Question Paper : 2319

Unique Paper Code : 62354343

GC-3

Name of the Paper : Analytical Geometry and Applied Algebra

Name of the Course : B.A. (Prog.) Mathematics (CBCS)

Semester : III

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any two parts from each question.

1. (a) Identify and sketch the curve :

$$(y - 3)^2 = 6(x - 2)$$

and also label the focus, vertex and directrix.

6

(b) Describe the graph of the curve :

$$4x^2 + y^2 + 8x - 10y = -13.$$

6

(c) Sketch the hyperbola :

$$16x^2 - y^2 - 32x - 6y = 57.$$

Also find the vertices, foci, asymptotes and the equation of directrices.

6

P.T.O.

2. (a) Find the equation of the parabola that has its focus at $(0, -3)$ and directrix $y = 3$. Also state the reflection property of parabola. 6
- (b) Find an equation for the ellipse with length of minor axis 8 and with vertices $(2, 6)$ and $(2, -4)$ and also sketch it. 6
- (c) Find and sketch the curve of the hyperbola that has vertices at $(2, 4)$ and $(10, 4)$ and foci are 10 units apart. 6

3. (a) Consider the equation :

$$3x^2 + 2\sqrt{3}xy + y^2 - 8x + 8\sqrt{3}y = 0.$$

Rotate the coordinate axes to remove the xy -term. Then identify the type of conic represented by the equation and sketch its graph. 6

- (b) Let an $x'y'$ -coordinate system be obtained by rotating an xy -coordinate system through an angle $\theta = 45^\circ$.

(i) Find the $x'y'$ -coordinate of the point whose xy -coordinates are $(\sqrt{2}, \sqrt{2})$.

(ii) Find an equation of the curve $x^2 - xy + y^2 - 6 = 0$ in $x'y'$ -coordinates. 6

- (c) Find the equation of the sphere through the four points $(4, -1, 2)$, $(0, -2, 3)$, $(1, -5, -1)$, $(2, 0, 1)$. 6

4. (a) Let $u = i - 3j + 2k$, $v = i + j$ and $w = 2i + 2j - 4k$. Find the length of $3u - 5v + 2w$. Also find the volume of the parallelepiped with adjacent edges u , v and w . $6\frac{1}{2}$

(b) Prove that :

$$u \cdot v = \frac{1}{4} \left(\|u + v\|^2 - \|u - v\|^2 \right). \quad 6\frac{1}{2}$$

(c) (i) Using vector, find the area of triangle with vertices A(2, 2, 0), B(-1, 0, 2) and C(0, 4, 3).

(ii) Sketch the surface $z = \cos x$ in 3-space. 3+3½

5. (a) (i) Find the parametric equation of line that is tangent to the circle $x^2 + y^2 = 25$ at the point (3, -4).

(ii) Show that the lines L_1 and L_2 intersect and find their point of intersection :

$$L_1 : x = 1 + 4t, \quad y - 3 = t, \quad z - 1 = 0$$

$$L_2 : x + 13 = 12t, \quad y - 1 = 6t, \quad z - 2 = 3t. \quad 3+3\frac{1}{2}$$

(b) Find the distance between the skew lines :

$$L_1 : x = 1 + 4t, \quad y = 5 - 4t, \quad z = -1 + 5t, \quad -\infty < t < \infty$$

$$L_2 : x = 2 + 8t, \quad y = 4 - 3t, \quad z = 5 + t, \quad -\infty < t < \infty. \quad 3+3\frac{1}{2}$$

(c) (i) Find the equation of the plane through (-1, 4, 2) that contains the line of intersection of the planes $4x - y + z - 2 = 0$ and $2x + y - 2z - 3 = 0$.

(ii) Do the points (1, 0, -1), (0, 2, 3), (-2, 1, 1) and (4, 2, 3) lie in the same plane.

Justify your answer.

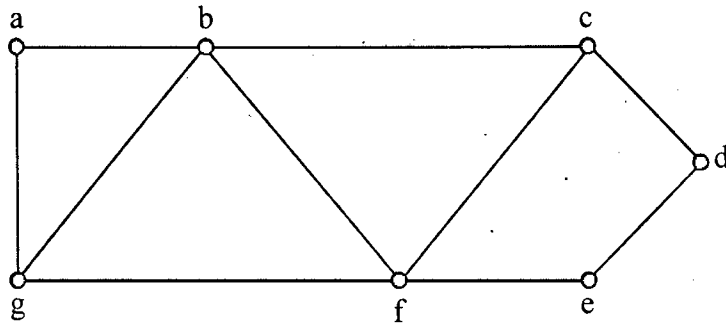
3+3½

P.T.O.

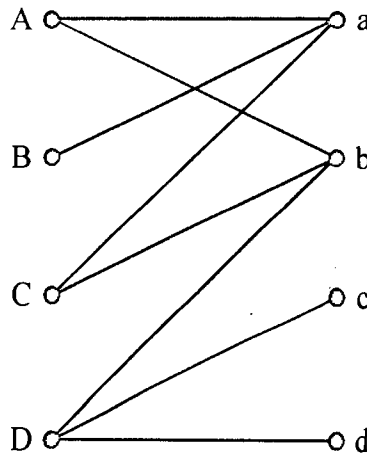
6. (a) Define a Latin square. Given an example of a Latin square of order 3. Is it unique ?
Justify. 6½

(b) Three Pitcher of sizes 7L, 4L and 3L (L = litre) are given. Only 7L pitcher is full.
Find a minimum sequence of pouring to make the quantity in three pitchers as
2L, 2L, 3L. 6½

(c) (i) In the following figure find all sets of two vertices whose removal disconnects
the graph :



(ii) Find a matching or explain why none exists for the following graph :



3+3½