[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2333 **GC-3** **Your Roll No.....**

Unique Paper Code : 62374311

Name of the Paper : Theory of Statistical Inference

Name of the Course : **B.A. (Program) STATISTICS (CBCS)**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Attempt **Six** questions in all.
3. Q. No. **1** is compulsory. Attempt **five** more questions.
4. Simple calculator can be used.

1. (a) Please identify True/False.

   (i) The Level of Significance is denoted by β.

   (ii) Normal distribution is a particular case of $\chi 2$ Distribution with one d.f.

   (iii) In the case of Poisson Distribution with parameter λ, $\bar{x}$ is Sufficient for λ.

   (iv) If T is consistent estimator of θ, then $aT+b$ is a consistent estimator of $a\theta+b$.

   (v) Mode of F-Distribution is $n_2 (n_1+2)/n_1(n_2+2)$.

   (vi) A MVUE has a variance that is as small or smaller than the variance of any other unbiased estimator. (1×6)

P.T.O.

(b) Discuss on the following :

(i) Null hypothesis and alternate hypothesis.

(ii) Mode of F-distribution.

(iii) Efficient estimators. (3×3)

2. (a) The manufacturer of television tubes knows from past experience that the average life of tubes is 2,000 hours with a standard deviation of 200 hours. A sample of 100 tubes has an average life of 1,950 hours. Test, at the 0.05 level of significance, if this sample came from a normal population of mean 2,000 hours. State your null and alternative hypothesis and indicate whether a one-tail or two-tail test is used and why ? Is the result of the test significant ?

(b) State and prove Invariance property of consistent estimator. (6,6)

3. (a) Find the limiting form of t-distribution for large n.

(b) X_1, X_2 and X_3 is a random sample of size 3 from a population with mean value μ and variance σ^2. T_1, T_2 and T_3 are the estimators used to estimate mean value μ, where

$$T_1 = X_1 + X_2 - X_3,\ T_2 = 2X_1 + 3X_3 - 4X_2 \text{ and } T_3 = \frac{1}{3}(AX_1 + X_2 + X_3)$$

(i) Are T_1 and T_2 unbiased estimators ?

(ii) For which value of A, T_3 is a consistent estimator ?

(iii) Which is the best estimator ? (6,6)

4. (a) Write p.d.f. of χ^2 distribution and also find the moment generating function of χ^2 distribution for n degree of freedom.

(b) Obtain 100(1-α)% confidence limits (for large samples) for the parameter λ of the poisson distribution :

$$f(x,\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!}; \qquad x = 0,1,2,\ldots \tag{6,6}$$

5. (a) Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type I error.

(b) For t-distribution with n degree of freedom, show that :

$$\mu_{2r+1} = 0$$

and

$$\mu_{2r} = n^{r} \frac{\Gamma\left(\frac{n}{2} - r\right)\Gamma\left(r + \frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{n}{2}\right)}; \quad r = 0, 1, 2, \ldots. \tag{4,8}$$

6. (a) Out of 8000 graduates in a town 800 are females; out of 1600 graduate employees 120 are females. Use χ^2 to determine if any distinction is made in appointment on the basis of sex. Value of χ^2 at 5% level for one degree of freedom is 3.84.

(b) Define MVU estimator. Show that MVU estimator is unique. (6,6)

7. (a) Given below are the gain in weights (in kgs.) of pigs fed on two diets A and B.

Diet A:	25	32	30	34	24	14	32	24	30	31	35	25			
Diet B:	44	34	22	10	47	31	40	30	32	35	18	21	35	29	22

Test, if the two diets differ significantly as regards their effect on increase in weight.

(b) Obtain Cramer-Rao lower bound for the variance of an unbiased estimator θ of normal distribution $N(\theta, \sigma^2)$, where σ^2 is known. (6,6)

8. Write short notes on any **three** of the following :

(i) Relation between F and t distribution.

(ii) Neyman-Pearson Lemma.

(iii) Maximum Likelihood estimator & its Properties.

(iv) Type I and Type II errors. (4,4,4)