[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 198 **G** **Your Roll No.....**

Unique Paper Code : 235351

Name of the Paper : Integration and Differential Equations

Name of the Course : **B.A. (Prog.) – Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Find the area enclosed by the curves $y^2 = 4x$ *and* $y = 2x - 4$. (6)

   (b) Obtain a reduction formula for $\int sec^n x\, dx$, n being a positive integer and hence evaluate $\int sec^6 x\, dx$. (6)

   (c) Evaluate :

$$\int \frac{1}{3sinx - 4cosx}\, dx\,.$$ (6)

2. (a) Show that

$$\int_0^{\frac{\pi}{2}} cos^m x cos\ nx dx = \frac{m}{m+n}\int_0^{\frac{\pi}{2}} \cos^{m-1} x cos(n-1)x dx\,.$$

   Further show that

$$\int_0^{\frac{\pi}{2}} \cos^n x\ cosnx\ dx = \frac{\pi}{2^{n+1}}\,,$$

   where m and n being positive integers. (6½)

P.T.O.

(b) Find the exact arc length of the curve

$$24xy = y^4 + 48 \text{ from } y = 2 \text{ to } y = 4. \quad (6\frac{1}{2})$$

(c) Let V_x and V_y be the volume of the solids that result when the region enclosed by

$$y = \frac{1}{x},\ y = 0,\ x = \frac{1}{2} \text{ and } x = b \ (b > 2)$$

is revolved about x – axis and y – axis, respectively. Is there any value of b for which $V_x = V_y$? (6½)

3. (a) Evaluate :

(i) $\int_0^{\pi/2} \log(\tan x + \cot x)\, dx$

(ii) $\int \frac{2x+3}{\sqrt{4x^2+5x+6}}\, dx$ (3+3)

(b) Solve :

$$y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0. \quad (6)$$

(c) Given that $y = x + 1$ is a solution of

$$(x+1)^2\frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 3y = 0.$$

Find a linearly independent solution by reducing the order. (6)

4. (a) Solve :

$$x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 4x - 6. \quad (6)$$

(b) Using the concept of Wronskian ,show that $e^x \sin x$ and $e^x \cos x$ are linearly independent solution of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0 .$$

Find the solution $y(x)$ satisfying the conditions $y(0) = 2$ and $y'(0) = -3$. (6)

(c) Assume that the population of a certain city increases at a rate proportional to the number of inhabitants at any time. If the population doubles in 40 years, in how many years will it triple ? (6)

5. (a) Using method of variation of parameters, find the general solution of

$$\frac{d^2y}{dx^2} + y = tanx .$$ (6½)

(b) Solve :

$$a^2 y^2 z^2 dx + b^2 x^2 z^2 dy + c^2 x^2 y^2 dz = 0.$$ (6½)

(c) Solve the system of equations :

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0$$

$$\frac{dy}{dt} + 5x + 3y = 0$$ (6½)

6. (a) (i) Classify the following partial differential equation into elliptic, parabolic or hyperbolic form :

$$r + 2s + t = 0$$

where $r = \frac{\partial^2 z}{\partial x^2}, s = \frac{\partial^2 z}{\partial x \partial y}, t = \frac{\partial^2 z}{\partial y^2}$. (3)

(ii) Eliminate the arbitrary function f to form the partial differential equation from the following equation

$$z = f\left(\frac{xy}{z}\right).$$ (3½)

(b) Find the general integral of the linear partial differential equation

$$z(xp - yq) = y^2 - x^2.$$ (6½)

(c) Find the complete integral of the partial differential equation

$$p + q = pq.$$ (6½)