[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 199 **G** **Your Roll No................**

Unique Paper Code : 235351

Name of the Paper : Integration and Differential Equations

Name of the Course : **B.A. (Prog.) – Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Find the area enclosed between the curves $y = x^2$, $y = \sqrt{x}$; $x = \frac{1}{4}$, $x = 1$. (6)

(b) Evaluate :

$$\int \frac{x^3}{\left(1+x^2\right)^{9/2}}\, dx$$ (6)

(c) If $I_n = \int_0^{\frac{\pi}{2}} \theta \sin^n\theta \; d\theta \;\; (n>1)$, show that

$$I_n = \frac{n-1}{n} I_{n-2} + \frac{1}{n^2}.$$

Deduce that $I_5 = \dfrac{149}{225}$ (6)

2. (a) Show that

$$\int_0^{\pi/2} Sin^n x \, dx = \frac{n-1}{n} \int_0^{\pi/2} Sin^{n-2} x \, dx \qquad (n \geq 2)$$

Use this result to derive Wallis Sine formulas :

$$\int_0^{\frac{\pi}{2}} Sin^n x\, dx = \frac{\pi}{2} \frac{1.3.5 \ldots\ldots (n-1)}{2.4.6 \ldots\ldots\ldots n} \; (n \text{ even and } \geq 2)$$

$$\int_0^{\frac{\pi}{2}} Sin^n x\, dx = \frac{2.4.6 \ldots\ldots (n-1)}{3.5.7 \ldots\ldots\ldots n} \; (n \text{ odd and } \geq 3)$$ (6½)

(b) Find the volume of the solid that results when the region above x – axis and below ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad (a > 0, b > 0)$$

is revolved about the x – axis. (6½)

(c) Find the circumference of a circle of radius a from the parametric equation

$x = a \cos t,\ y = a \sin t \quad (0 \leq t \leq 2\pi)$. (6½)

3. (a) Evaluate :

(i) $\int_0^{\pi/2} \log \sin x\, dx$

(ii) $\int \frac{x+2}{\sqrt{4+3x-2x^2}}\, dx$ (3+3)

(b) Solve :

$$(x^2 + y^2 + x)\, dx + xy\, dy = 0.$$ (6)

(c) Given that $y = x$ is a solution of

$$\left(x^2+1\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0.$$

Find a linearly independent solution by reducing the order. (6)

4. (a) Solve :

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=4\ sin\ log\ x \quad (6)$$

(b) Using the concept of Wronskian, show that e^x and e^{2x} are linearly independent solution of

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=0.$$

Find the solution $y(x)$ satisfying the conditions $y(0) = 0$ and $y'(0) = 1$. (6)

(c) Find the orthogonal trajectories of the family of circles

$$x^2 + y^2 = c^2. \quad (6)$$

5. (a) Using method of variation of parameters, find the general solution of

$$\frac{d^2y}{dx^2}+y=cot\ x. \quad (6½)$$

(b) Solve :

$$(yz + z^2)dx - xzdy + xydz = 0 \quad (6½)$$

(c) Solve the system of equations :

$$\frac{dx}{dt}+\frac{dy}{dt}-y=2t+1$$

$$2\frac{dx}{dt}+2\frac{dy}{dt}+y=t \quad (6½)$$

6. (a) (i) Form a partial differential equation by eliminating the constant a and b from the following equation :

$$2z = (ax + y)^2 + b.$$

(ii) Classify the following partial differential equation into hyperbolic, parabolic or elliptic form :

$$(n-1)^2 \frac{\partial^2 z}{\partial x^2} - y^{2n} \frac{\partial^2 z}{\partial y^2} = ny^{2n-1} \frac{\partial z}{\partial y}.$$ (3+3½)

(b) Find the general integral of the following partial differential equation

$$y^2 p - xy\, q = x(z-2y)$$ (6½)

(c) Find the complete integral of the partial differential equation

$$(p^2 + q^2)y = qz.$$ (6½)