[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 202 **G** **Your Roll No.....**

Unique Paper Code : 236351

Name of the Paper : MATHEMATICAL PROGRAMMING

Name of the Course : **B.A. Programme – Operational Research**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** questions.
3. **All** questions carry equal marks.
4. Statistical Tables can be used.

1. (a) What is Integer programming problem ? How does it differ from linear programming problem ? (3)

   (b) Determine whether the given function is concave, convex or neither;

   $$f(x_1, x_2, x_3) = -x_1^2 - x_2^2 - 2x_3^2 + 0.5x_1x_2$$

   where, $x = (x_1, x_2, x_3) \in R^3$ (5)

   (c) Consider the following constraint optimization problem;

   $$\max\ f(x) = \ln(x_1 + 1) - x_2^2$$

P.T.O.

s.t. $x_1 + 2x_2 \leq 3, \quad x_1, x_2 \geq 0$

use KKT conditions to derive the optimum solution. (7)

2. (a) Use the branch and bound technique to solve the following Integer programming problem

Maximize $z = x_1 + x_2$

Subject to; $2x_1 + 5x_2 \leq 16$

$6x_1 - 5x_2 \leq 30$

$x_1, x_2 \geq 0$, and integer.

Given non integral solution is :

$x_1 = 7/2, x_2 = 9/5$ Maximum $Z = 53/10$ (10)

(b) What is meant by **Zero-One programming problem** ? Explain it with the help of an appropriate example. (5)

3. (a) Let $f(x)$ be differentiable in its domain. If $f(x)$ is defined on an open convex set S, then show that $f(x)$ is convex if

$$f(x_2) - f(x_1) \geq (x_2 - x_1)^T \nabla f(x)$$

for all $x_1, x_2 \in S$. (8)

(b) The production function of a commodity is given by :

$Q = 40F + 3F^2 + (F^3/3)$ where Q is the total output and F is the unit of inputs.

Find the number of units of input required to give the maximum output. (7)

4. (a) Write the Necessary and Sufficient condition for optimizing a multivariable un-constrained problem in non linear programming. Determine the relative maximum and minimum (if any) of the following function :

$$f(X) = x_1 + 2x_2 + x_2x_3 - x_1^2 - x_2^2 - x_3^2 .$$ (9)

(b) Derive the Necessary and Sufficient condition for optimizing a two variables non-linear optimization problem with single equality constraint by Lagrangian multiplier technique. (6)

5. (a) Define Quadratic programming. Derive the Kuhn-Tucker conditions for Quadratic programming problem. (8)

(b) Solve the following non-linear programming problem, using the Lagrangian multiplier method.

Optimize $Z = 4x_1^2 + 2x_3^2 - 4x_1x_2$

Subject to $x_1 + x_2 + x_3 = 15$

$2x_1 - x_2 + 2x_3 = 20$

$x_1, x_2, x_3 \geq 0$ (7)

6. Solve the following quadratic programming problem using Wolf's method

Maximize $z = 15x_1 + 30x_2 + 4x_1x_2 - 2x_1^2 - 4x_2^2$

Subject to $x_1 + 2x_2 \leq 30$

$x_1, x_2 \geq 0$ (15)

7. Write down short notes on :–

(a) Gomory's cutting plane method. (5)

(b) Bordered Hessian matrix. (5)

(c) convex and concave functions. (5)