

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 776-A C Roll No.....

Unique Paper Code : 235451

Name of the Course : B.A. (Prog.)

Name of the Paper : Anal. Geom. & Appl. Alg.

Semester : IV

Duration : 3 Hours Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any Two parts from each question.

1. (a) Identify and sketch the curve

$$y^2 - 6y - 2x + 1 = 0$$

and also label the focus, vertex and directrix. (6)

- (b) Identify and sketch the conic represented by the equation

$$4x^2 + y^2 + 8x - 10y + 13 = 0$$

and also label the foci, the vertices and the ends of the major axis. (6)

- (c) Find the center, vertices, foci and asymptotes of the hyperbola whose equation is

$$x^2 - y^2 - 4x + 8y - 21 = 0$$

and sketch its graph. (6)

2. (a) Find an equation for the parabola whose vertex is (1, 1) and directrix  $y = -2$ . Also state reflection properties of Parabola. (6)

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- (b) Find an equation for the ellipse with foci  $(1, 2)$  and  $(1, 4)$ ; minor axis of length 2 and sketch its graph. (6)
- (c) Find an equation for the hyperbola that passes through the origin and whose asymptotes are  $y = 2x + 1$  and  $y = -2x + 3$ . (6)
3. (a) Let an  $x'y'$ -coordinate system be obtained by rotating an  $xy$ -coordinate system through an angle  $\theta = 60^\circ$ .
- (i) Find the  $x'y'$ -coordinate of the point whose  $xy$ -coordinates are  $(-2, 6)$ .
- (ii) Find an equation of the curve  $\sqrt{3}xy + y^2 = 6$  in  $x'y'$ -coordinates. (6)
- (b) Rotate the coordinate axes to remove the  $xy$ -term of the curve  $x^2 + 4xy - 2y^2 - 6 = 0$ , then name the conic and sketch its graph. (6)
- (c) Find the angle between a diagonal of a cube and one of its edges. (6)
4. (a) Find equation of two spheres that are centered at the origin and are tangent to the sphere of radius 1 centered at  $(0, 0, 7)$ . (6½)
- (b) (i) If for any two vectors  $\mathbf{u}$  and  $\mathbf{v}$ ,
- $$\mathbf{u} + \mathbf{v} = \|\mathbf{u} + \mathbf{v}\| \mathbf{i}$$
- then prove that  $\mathbf{u}$  is orthogonal to  $\mathbf{v}$ . Interpret this result geometrically.
- (ii) Find the angle between the vectors  $\mathbf{u} = \hat{i} - 2\hat{j} + 2\hat{k}$  and  $\mathbf{v} = -3\hat{i} + 6\hat{j} + 2\hat{k}$ . (3,3½)
- (c) Consider the parallelepiped with adjacent vectors
- $$\mathbf{u} = 3\hat{i} + 2\hat{j} + \hat{k}, \mathbf{v} = \hat{i} + \hat{j} + 2\hat{k}, \mathbf{w} = \hat{i} + 3\hat{j} + 3\hat{k}$$
- (i) Find the volume of the parallelepiped.
- (ii) Find the area of the face determined by  $\mathbf{u}$  and  $\mathbf{v}$ . (6½)

5. (a) Find the distance between the skew lines

$$L_1: x = 1 + 7t, \quad y = 3 + t, \quad z = 5 - 3t, \quad -\infty < t < \infty$$

$$L_2: x = 4 - t, \quad y = 6, \quad z = 7 + 2t, \quad -\infty < t < \infty \quad (6\frac{1}{2})$$

(b) (i) Find the parametric equation of the line passing through the point  $(-2, 0, 5)$  that is parallel to the line

$$x = 1 + 2t, \quad y = 4 - t, \quad z = 6 + 2t.$$

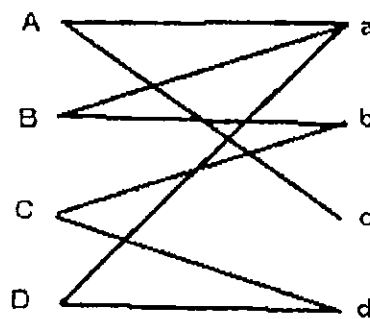
(ii) Find the equation of the plane passing through the points  $P(-2, 1, 1)$ ,  $Q(0, 2, 3)$  and  $R(1, 0, -1)$ . (3.3\frac{1}{2})

(c) (i) Find the equation of the plane through the point  $(1, 4, 2)$  that contains the line of intersection of the planes  $4x + y + z = 2$  and  $2x + y - 2z - 3 = 0$ .

(ii) Find the distance from the point  $(1, 1, 3)$  and the plane  $3x + 2y + 6z = 6$ . (3.3\frac{1}{2})

6. (a) A chemical company wants to test a chemical reaction using seven different levels of catalyst a, b, c, d, e, f, g. In the manufacturing process the raw material comes from the previous stage in batches and the catalyst must be added immediately. If there are seven reactors A, B, C, D, E, F, G; in which the catalytic reaction can take place. Show how to design the experiment using seven batches of raw material so as to minimize the effect of the different batches and of different reactors. (Use Latin square) (6\frac{1}{2})

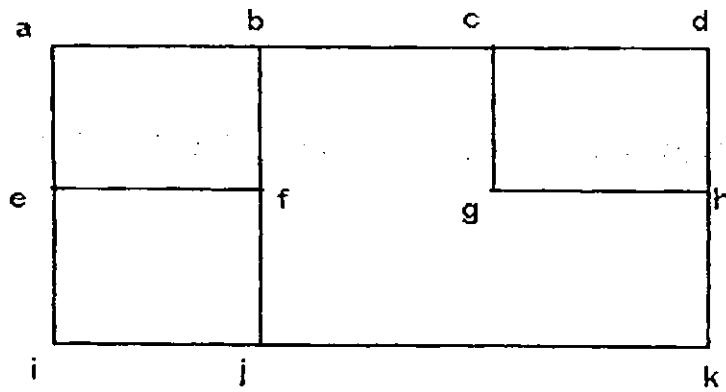
(b) Find a matching or explain why none exists for the following graph :



(6\frac{1}{2})

P.T.O.

- (c) Find all sets of two vertices whose removal disconnects the remaining graph in the following figure



(6½)