[This question paper contains 4 printed pages.]

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All questions are compulsory.

3. Attempt any Two parts from each question.

1. (a) Identify and sketch the curve

    $y^2 - 8x - 6y - 23 = 0$

    and also label the focus, vertex and directrix.   (6)

   (b) Sketch the ellipse

    $9x^2 + 4y^2 + 18x - 24y + 9 = 0$

    and also label the foci, the vertices and the ends of the major axis.   (6)

   (c) Describe the graph of the hyperbola

    $4x^2 - 9y^2 + 16x + 54y - 29 = 0$

    and sketch its graph.   (6)

2. (a) Find an equation for the parabola whose axis is $y = 0$ and passes through (3, 2) and (2, -3). Also sketch its rough graph showing the reflection property of parabola at the point (2, -3).   (6)

(b) Find an equation for the ellipse with length of minor axis 8 and with vertices (2, 6) and (2, −4) and also sketch it. (6)

(c) Find an equation for a hyperbola whose vertices are (±1,0) and asymptotes are $y = \pm 2x$. (6)

3. (a) Rotate the coordinate axes to remove the xy-term of the curve xy = 1, then name the conic and sketch its graph. (6)

(b) (i) Find the vector of length 2 that makes an angle $\dfrac{\pi}{4}$ with the positive x-axis.

(ii) Find a unit vector which has the same direction as the vector from the point P(1, 0, 2) to the point Q(3, 1, 1). (3,3)

(c) (i) Sketch the graph of z = sin y in 3-space.

(ii) Find the orthogonal projection of vector $v = \hat{i} + \hat{j} + \hat{k}$ on $b = 2\hat{i} + 2\hat{j}$. (3,3)

4. (a) A sphere S has center in the first octant and is tangent to each of the three co-ordinate planes. The distance from the origin to the sphere is $3 - \sqrt{3}$ units. What is the equation of the sphere ? (6½)

(b) If u and v are non-zero vectors in 3-space and if θ is the angle between them, then show that

$$\cos\theta = \frac{u \cdot v}{\|u\| \|v\|}$$ (6½)

(c) Determine whether the vectors u = <1,−2, 1>; v = <3, 0,−2> and w = <5,−4, 0> lie in the same plane ? Also find u × v and show that it is orthogonal to u. (6½)

5. (a) Find the parametric equations of the line L passing through the points P(2, 4, –1) and Q(5, 0, 7). Where does the line intersect the xz plane ? (6½)

   (b) (i) Show that the lines L_1 and L_2 intersect and find their point of intersection.

   $L_1 : x = 2 + t \quad y = 2 + 3t \quad z = 3 + t$

   $L_2 : x = 2 + t \quad y = 3 + 4t \quad z = 4 + 2t$

   (ii) Where does the line $x = 1 + 3t, \; y = 2 - t$ intersect

   (a) the x-axis   (b) the y-axis. (3½,3)

   (c) (i) Find an equation of the plane passing through the point P(–3, 0, 7) and perpendicular to the vector $n = 5\hat{i} + 2\hat{j} - \hat{k}$.
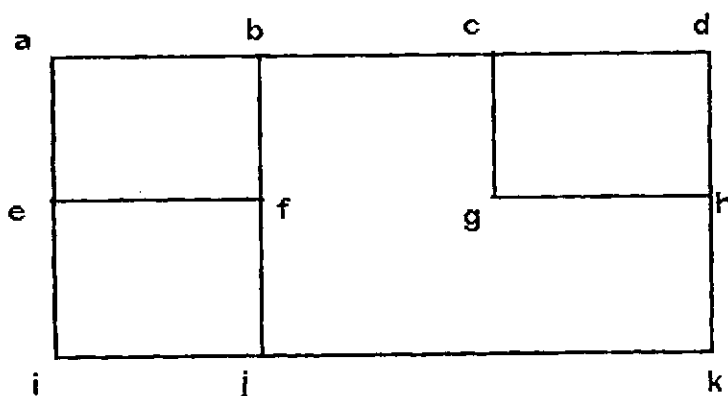
   (ii) Determine whether the line

   $x = 4 + 2t \quad y = -t \quad z = -1 - 4t$

   is parallel or perpendicular to the plane $3x + 2y + z - 7 = 0$. (3,3½)

6. (a) Construct a Latin square of order 5 on {0, 1, 2, 3, 4}. (6½)

   (b) In the following figure, find all sets of three corners that have all 11 corners under surveillance. Give a careful logical analysis.
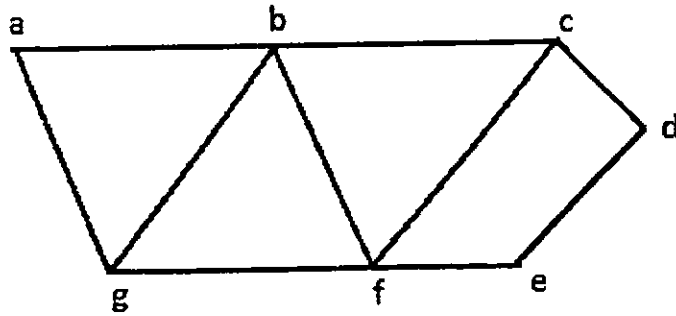

   (6½)

(c) In the following figure find

    (i) All sets of two vertices whose removal disconnects the graph.

    (ii) All sets of two edges whose removal disconnects the graph.

(6½)

(2000)