This question paper contains 3 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **782**

Unique Paper Code : **237451** **C**

Name of the Paper : **Statistical Inference and Regression Analysis**

Name of the Course : **B.A. (Program)**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all.

Question No. **1** is compulsory.

1. (*i*) Give examples of estimators which are :

   (*a*) Consistent and unbiased

   (*b*) Consistent but not unbiased.

   (*ii*) Obtain sufficient estimator for the parameter θ of the distribution :

$$f(x : \theta) = \theta . e^{-\theta x}, \quad x > 0, \theta > 0$$

   (*iii*) Show by means of an example that MLE need not be unique.

   (*iv*) Let p be the probability of appearance of head in a single toss of coin in order to test :

   $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of Type I error and Type II error.

(v) For the simple regression model :

$Y = \beta_0 + \beta_1 X + \varepsilon$, show that $\sum e_i Y_i = 0$. 5×3=15

2. (a) State and prove invariance property of consistent estimators. Hence or otherwise, obtain consistent estimator for $\frac{1}{\theta}$ of Poisson population with parameter θ.

(b) State Rao-Blackwell theorem and explain its significance. 8,4

3. (a) Define MVU estimator. Show that MVU estimator is unique.

(b) Obtain Cramer-Rao lower bound for the variance of an unbiased estimator θ of normal distribution $N(\theta, \sigma^2)$, where σ^2 is known. 6,6

4. (a) Obtain MLE of θ for a population with p.d.f. :

$$f(x : \theta) = (1 + \theta)\, x^{\theta},\ 0 < x < 1$$

based on a random sample of size n. Also verify whether there exists a sufficient statistic for θ.

(b) Differentiate between point and interval estimations. Obtain 100 $(1 - \alpha)$% confidence interval for the parameter θ of the normal distribution $N(\theta, \sigma^2)$, where σ^2 is known. 6,6

5. (a) Explain the following :

(i) Simple hypothesis and power of a test

(ii) Critical region and level of significance

(iii) Neymann-Pearson lemma.

(b) Discuss run test for randomness of a series. 6,6

6. Write short notes on any *three* :

   (*i*) Properties of ML estimators

   (*ii*) Sign test

   (*iii*) Confidence interval for proportions

   (*iv*) Multiple linear regression. 4,4,4

7. Discuss the analysis of variance for simple linear regression model using matrix approach. 12

8. Obtain the BLUE of $\hat{\beta}_1$ for simple linear regression model : 12

$$Y = \beta_0 + \beta_1 X + \varepsilon.$$