[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5384 **D** **Your Roll No...............**

Unique Paper Code : 235451

Name of the Course : **B.A. (Prog.)**

Name of the Paper : Analysic Geometry & Applied Algebra

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **Two** parts from each question.

1. (a) Sketch the parabola

$$x = y^2 - 4y + 2$$

and label the focus, vertex and directrix. (6)

(b) Describe the graph of the equation

$$16x^2 + 9y^2 - 64x - 54y + 1 = 0. \quad (6)$$

(c) Find the center, vertices, foci and asymptotes of the hyperbola whose equation is

$$4y^2 - x^2 + 40y - 4x + 60 = 0$$

and sketch its graph. (6)

2. (a) Find an equation for the parabola whose vertex is (5, –3); axis parallel to y-axis and passes through the point (9, 5). (6)

(b) Find an equation for the ellipse with end points of major axis (±6, 0) and passes through (2, 3). Also sketch its rough graph showing the reflection property of ellipse at the point (2, 3). (6)

(c) Find an equation for the hyperbola with foci (1, 8) and (1, −12) and whose vertices are 4 units apart. (6)

3. (a) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle $\theta = 30°$.

(i) Find the $x'y'$-coordinate of the point whose xy-coordinates are $(1, -\sqrt{3})$.

(ii) Find an equation of the curve $2x^2 + 2\sqrt{3}\,xy = 3$ in $x'y'$-coordinates. (6)

(b) Rotate the coordinate axes to remove the xy-term of the curve

$$x^2 + 2\sqrt{3}\,xy + 3y^2 + 2\sqrt{3}\,x - 2y = 0,$$

then name the conic. (6)

(c) (i) Find the vector of length 4 that makes an angle $\frac{\pi}{6}$ with the positive x-axis.

(ii) Find the angle that the vector $v = -\hat{i} + \sqrt{3}\,\hat{j}$ makes with the positive x-axis. (3,3)

4. (a) Find the equation of the sphere that has (1, −2, 4) and (3, 4, −12) as end points of a diameter. (6½)

(b) (i) Find the direction cosines of the vector $v = 2\hat{i} + 3\hat{j} - 6\hat{k}$ if it makes angles α, β, γ with the x-axis, y-axis, z-axis respectively and show that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

(ii) For any two vectors u and v, prove that

$$u.v = \frac{1}{4}\|u + v\|^2 - \frac{1}{4}\|u - v\|^2$$ (3,3½)

(c) (i) Show that if u and v are vectors in 3-space, then

$$\|u \times v\|^2 = \|u\|^2 \|v\|^2 - (u.v)^2$$

(ii) Use a scalar triple product to determine whether the vectors lie in the same plane $u = 5\hat{i} - 2\hat{j} + \hat{k}$, $v = 4\hat{i} - \hat{j} + \hat{k}$, $w = \hat{i} - \hat{j}$. (3,3½)

5. (a) Let L_1 and L_2 be the lines whose parametric equations are

$L_1 : x = 1 + 2t \quad y = 2 - t \quad z = 4 - 2t$

$L_2 : x = 9 + t \quad y = 5 + 3t \quad z = -4 - t$

(i) Show that the lines L_1 and L_2 intersect at the point (7, −1, −2).

(ii) Find the acute angle between L_1 and L_2 at their point of intersection. (6½)

(b) (i) Find the parametric equation of the line L passing through the points (2, 4, −1) and (5, 0, 7). Where does the line intersect the xy plane.

(ii) Determine whether the line

$x = 3 + 8t \quad y = 4 + 5t \quad z = -3 - t$

is parallel to the plane $x - 3y + 5z = 12$. (3,3½)

(c) (i) Find the equation of the plane through the points (1, 2, −1), (2, 3, 1) and (3, −1, 2).

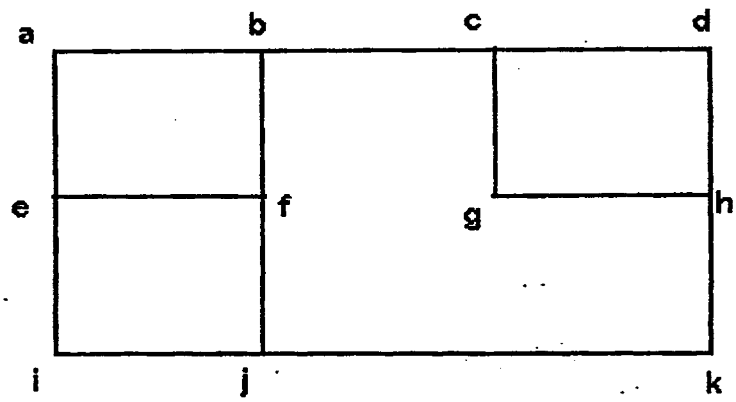
(ii) Find the distance between the planes $x + 2y - 3z = 3$ and $2x + 4y - 4z = 7$. (3,3½)

6. (a) Show that the given latin square cannot be obtained from a group table :

A	B	C	D	E
B	A	E	C	D
C	D	A	E	B
D	E	B	A	C
E	C	D	B	A

(6½)

(b) The following figure represents a section of city's street map. We want to position police at corners (vertices) so that they can keep every block (edge) under surveillance i.e. every edge should have a policeman at least one of its corner. What is the smallest number of police that can do this job. (6½)



(c) Find a matching or explain why none exists for the following graph : (6½)