[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5383 **D** **Your Roll No.....**

Unique Paper Code : 235451

Name of the Course : **B.A. (Prog.)**

Name of the Paper : Analytic Geometry & Applied Algebra

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **Two** parts from each question.

1. (a) Identify and sketch the curve

$$x^2 - 4x + 2y = 1.$$ (6)

(b) Sketch the conic represented by the equation

$$x^2 + 9y^2 + 2x - 18y + 1 = 0$$

and also label the foci, the vertices and the ends of the minor axis. (6)

(c) Sketch the hyperbola

$$4y^2 - x^2 + 40y - 4x + 60 = 0$$ (6)

2. (a) Find an equation for the parabola whose focus is $(-1, 4)$ and directrix $x = 5$ and sketch its graph. (6)

(b) Find an equation for the ellipse with length of major axis 6 and with foci $(2, 1)$ and $(2, -3)$. Also state reflection property of ellipse. (6)

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(c) Find the equation of the hyperbola with vertices $(0, \pm 8)$ and asymptotes are $y = \pm \frac{4}{3} x$. Also sketch its graph. (6)

3. (a) Rotate the coordinate axis to remove the xy-term of the curve $x^2 - xy + y^2 - 2 = 0$, then name the conic. (6)

(b) (i) Find the component of the vector v in 2-space having $\| v \| = 5$ and $\theta = 120°$.

(ii) Find the angle that the vector $v = -\sqrt{3}\,\hat{i} + \hat{j}$ makes with the positive x-axis. (3,3)

(c) (i) Sketch the graph of $x^2 + z^2 = 1$ in 3-space.

(ii) Find the orthogonal projection of vector

$v = 6\hat{i} + 3\hat{j} + 2\hat{k}$ on $b = \hat{i} - 2\hat{j} - 2\hat{k}$. (3,3)

4. (a) Find the equations of two spheres that are centered at the origin and are tangent to the sphere of radius 1 centered at (3,–2,4). (6½)

(b) Use vectors to show that A(2, –1, 1), B(3, 2, –1) and C(7, 0, –2) are vertices of a right triangle. At which vertex is the right angle ? (6½)

(c) Let u and v be non-zero vectors in 3-space and let θ be the angle between these vectors and are positioned so that their initial points coincide, then show that

$$\|u \times v\| = \|u\| \cdot \|v\| \sin \theta \qquad (6½)$$

5. (a) Let L_1 and L_2 be the lines

$L_1 : x = 1 + 4t \quad y = 5 - 4t \quad z = -1 + 5t$

$L_2 : x = 2 + 8t \quad y = 4 - 3t \quad z = 5 + t$

(i) Are the lines parallel ?

(ii) Do the lines intersect ? (6½)

(b) (i) Find the parametric equations of the line passing through the point P(4, 1, 2) and parallel to the vector $\hat{i} - \hat{j}$.

(ii) Where does the line

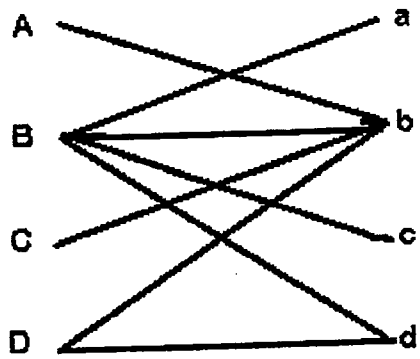
$x = 1 + t, \quad y = 3 - t, \quad z = 2t$

intersects the cylinder $x^2 + y^2 = 16$. (3,3½)

(c) Find the equation of the plane through the point (–1, 2, –5) and perpendicular to the planes $2x - y + z = 1$ and $x + y - 9z = 3$. (6½)

6. (a) A supermarket wishes to test the effect of putting cereal on four shelves at different heights. Show how to design such an experiment lasting four weeks and using four brands of cereal. (6½)

(b) Find a matching or explain why none exists for the following graph : (6½)



(c) What are the other sets of 2 edges whose removal disconnects the graph in the following figure besides (a, b), (a, e) and (c, d), (d, h) ? Either produce other or give an argument why no other exists. (6½)