[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5390 **D** **Your Roll No...............**

Unique Paper Code : 237451

Name of the Course : **B.A. (Program)**

Name of the Paper : Statistical Inference and Regression Analysis

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all.
3. Question No. **1** is compulsory.

1. (i) If T is an unbiased estimator for θ. Examine whether T^2 is unbiased for θ^2.

(ii) For a random sample of size n drawn from a population with p.d.f. :

$f(x, \theta) = \theta x^{\theta-1}$; $0 < x < 1$, $\theta > 0$.

Show that $\prod_{i=1}^{n} X_i$ is sufficient for θ.

(iii) Examine whether there exist an MVB estimator for θ in Cauchy's population with p.d.f. :

$$f(x, \theta) = \frac{1}{\pi} \cdot \frac{1}{1+(x-\theta)^2}, \quad -\infty < x < \infty$$

(iv) The probability that a given die shows even number is p. To test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{1}{3}$, following procedure is adopted. Toss the die twice and accept H_0 if both times it shows even number. Find probabilities of Type I and Type II errors.

(v) For the simple regression model $Y = \beta_0 + \beta_1 X + \varepsilon$, show that $\sum e_i X_i = 0$.

(3×5=15)

2. (a) Let $\{T_n\}$ be sequence of estimators such that for all $\theta \in \Theta$.

   (i) $E_\theta(T_n) \to \gamma(\theta)$ as $n \to \infty$

   (ii) $Var_\theta\ (T_n) \to 0$ as $n \to \infty$

   Prove that T_n is a consistent estimator of $\gamma(\theta)$.

   (b) Find the MLE for the parameter λ of a Poisson distribution on the basis of a sample of size n. Also find its variance. (7,5)

3. (a) Let X_1, X_2, and X_3 be a random sample from a population with mean μ and variance σ^2. If $T_1 = X_1 + X_2 - X_3$, $T_2 = 2X_1 - 4X_2 + 3X_3$ and

   $$T_3 = \frac{1}{3}(\lambda X_1 + X_2 + X_3).$$

   (i) Show that T_1 and T_2 are unbiased estimators of μ.

   (ii) Find the value of λ such that T_3 is unbiased for μ.

   (iii) Which is the best estimator among T_1, T_2 and T_3?

   (b) Suppose T_1 is a minimum variance unbiased estimate of $\gamma(\theta)$ and T_2 is any other unbiased estimate of $\gamma(\theta)$ with variance σ^2/e. Then prove that the correlation between T_1 and T_2 is $\sqrt{e}$. (6,6)

4. (a) Obtain the MVB estimator for θ in a normal population $N(\theta, \sigma^2)$, where σ^2 is known.

(b) Obtain 100(1-α)% confidence interval for the parameter θ of the normal distribution $N(\theta, \sigma^2)$, where σ^2 is unknown. (6,6)

5. (a) State Neyman-Pearson Lemma and explain its significance.

(b) Define level of significance and power of test.

Let $f(x, \theta) = \frac{1}{\theta} e^{\left(\frac{-x}{\theta}\right)}, \ 0 < x < \infty, \ \theta > 0$

To test $H_0 : \theta = 2$ against $H_1 : \theta = 1$, use random sample x_1 and x_2 of size 2 for the critical region $W = \{(x_1, x_2) : 9.5 \leq x_1 + x_2\}$

Find (i) level of significance

(ii) Power of test (5,7)

6. Write short notes on any **three** of the following :

(a) Confidence interval for proportion

(b) Run Test

(c) Properties of Maximum likelihood estimator

(d) Rao-Blackwell Theorem (4,4,4)

7. For simple linear regression model $y = \beta_0 + \beta_1 X + \varepsilon$

(i) Obtain the estimates of the parameters β_0, β_1 using principle of least squares.

(ii) Obtain the variances of the estimates $\widehat{\beta_0}$, $\widehat{\beta_1}$. (6,6)

8. (i) Write a short note on multiple linear regression model and explain its significance.

(ii) State Gauss - Markov theorem and explain its importance. (6,6)