Sr. No. of Question Paper : 241   E   Your Roll No................

Unique Paper Code   : 235451

Name of the Course   : **B.A. (Prog.) Mathematics**

Name of the Paper   : Analytical Geometry and Applied Algebra

Semester   : IV

Duration : 3 Hours   Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question.

1. (a) Sketch the parabola whose equation is

$$4y^2 - x + 8y + 5 = 0$$

Also label the focus, vertex, and directrix.   (6½)

(b) Identify the sketch the conic represented by the equation.

$$x^2 + 4y^2 - 10x - 24y + 45 = 0$$

and also label the foci, the vertices and the ends of minor axis.   (6½)

(c) Sketch the hyperbola whose equation is

$$16x^2 - 25y^2 - 32x + 50y - 409 = 0$$

and also label the vertices, foci and asymptotes.   (6½)

2. (a) Find an equation for the parabola whose focus is $(3, 0)$ and directrix $x = -2$. (6)

(b) Find an equation for the ellipse whose foci are $(2, 1)$ and $(2, -3)$ and major axis of length 6. (6)

(c) Find an equation for a hyperbola with vertices $(0, \pm 3)$ and foci $(0, \pm 5)$. Also state reflection property of hyperbolas. (6)

3. (a) Suppose that the axis of an xy-coordinate system are rotated through an angle of $\theta = 30°$ to obtain an x^1y^1-coordinates system. Find the equation of the curve

$$x^2 - xy + y^2 - 6 = 0$$

in x^1y^1-coordinates. (6½)

(b) Sketch the graph of $y = \sin x$ in 3-space. (6½)

(c) Let $u = 2i - 3j + 5k$, $v = 2i + j$ and $w = 2i + 2j - 4k$ find

(i) $\|3u - 5v + w\|$

(ii) $\|-2u\| + 2\|v\|$ (3,3½)

4. (a) Show that if $\vec{u}$ and $\vec{v}$ are vectors in 3-space, then

$$\|\vec{u} \times \vec{v}\| = \|\vec{u}\|^2 \|v\|^2 - \left(\vec{u}.\vec{v}\right)^2 .$$
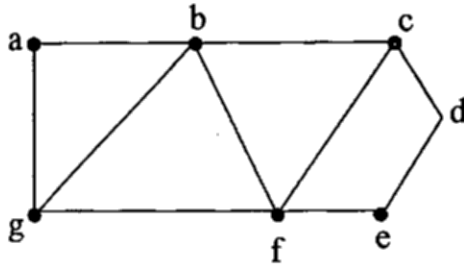
(6)

(b) Find the angle between a diagonal of a cube and one of its edges. (6)

(c) Prove that the area A of the parallelogram that has $\vec{u}$ and $\vec{v}$ as adjacent

sides is $A = \|\vec{u} \times \vec{v}\|$. Also find the area of the parallelogram that has

$\vec{u}(1,-1,2)$ and $\vec{v}(0,3,1)$ as adjacent sides. (6)

5. (a) Find parametric equations for the line segment joining the points $P_1(2,4,-1)$
and $P_2(5,0,7)$. (6½)

(b) Show that the lines

x = −2 + t, y = 3 + 2t, z = 4 − t

x = 3 − t, y = 4 − 2t, z = t

are parallel and find an equation of the plane they determine. (6½)

(c) Find the distance between the given planes which are

−2x + y + z = 0 and 6x − 3y − 3z − 5 = 0 (6½)

6. (a) Construct a Latin square of order 5 on {A, B, C, D, E}. (6)

(b) In the following figure find all sets of three corners that have 11 corners under surveillance, give a careful logical analysis : (6)
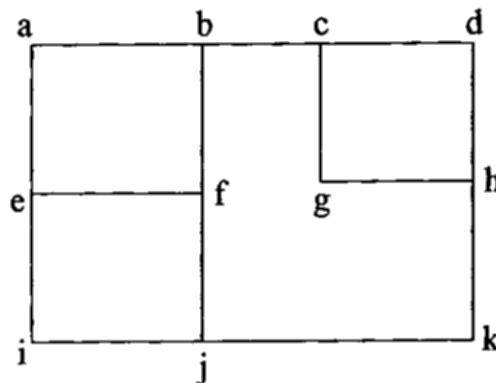
(c) In the following figure find :

(i) All sets of two vertices whose removal disconnects the graph.

(ii) All sets of two edges whose removal disconnects the graph.

(6)

(200)