[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 246 **E** **Your Roll No...............**

Unique Paper Code : 237451

Name of the Course : **B.A. (Program)**

Name of the Paper : Statistical Inference and Regression Analysis

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **six** questions.
3. Question No. **1** is compulsory.
4. Use of Simple Calculator is allowed.

1. (a) Let X follow $N(\mu,\sigma^2)$ where σ^2 is known. Show that X is a consistent estimator of μ for large n.

   (b) Show by means of an example that MLE need not be unbiased.

   (c) Obtain sufficient estimator for the parameter θ of $f(x,\theta) = \theta x^{\theta-1}\ 0 < x < 1$.

   (d) For the simple Linear Regression Model $Y = \beta_0 + \beta_1 x + \varepsilon_1$

   show that $\sum e_i \hat{Y}_i = 0$.

   (e) Let p be the probability that a coin will fall head in a single toss. In order to test

   $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{2}{3}$

   the coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of Type I error and Type II error.

   (2,2,2,2,2)

2. (a) State sufficient conditions for consistency. Obtain consistent estimator for parameter θ of the Bernoulli population with pmf

$$f(x,\theta) = \begin{cases} \theta^x (1-\theta)^{1-x} & x = 0,1 \\ 0, & \text{otherwise.} \end{cases}$$

P.T.O.

(b) Show that MVUE is unique. (6,6)

3. (a) Given one observation from a population with pdf $f(x, \theta) = \frac{2}{\theta^2}(\theta - x), 0 \le x \le \theta$, obtain $100(1-\alpha)\%$ confidence interval for θ.

(b) Obtain MVB estimator for parameter θ of Poisson distribution with pmf

$$f(x,\theta) = \begin{cases} \frac{e^{-\theta}\theta^x}{x!}, & x = 0,1,2 \ldots.. \\ 0, & \text{otherwise.} \end{cases}$$

(7,5)

4. (a) State Neyman-Pearson Lemma and define the following terms :
   (i) Simple Hypothesis
   (ii) Critical Region
   (iii) Two types of errors

(b) Let $X \sim N(\mu, 4)$, μ unknown. To test $H_0 : \mu = -1$ against $H_1 : \mu = 1$ based on a sample of size 10 from this population, we use the critical region: $x_1 + 2x_2 + \ldots\ldots.10x_{10} \ge 0$. What is its size ? You may use one of these values as answers and justify your answer. (0.75,0.5,0.1808). (6,6)

5. (a) State Rao – Blackwell Theorem and explain its significance.

(b) Define MVU estimator. Show that MVUE is unique. (6,6)

6. Discuss the analysis of variance for simple linear regression model using matrix form. (12)

7. For the simple Linear Regression Model $Y = \beta_0 + \beta_1 x + \varepsilon$,

   (i) Prove that $\text{cov}\left(\widehat{\beta_0}, \widehat{\beta_1}\right) = -\frac{\bar{x}\sigma^2}{s_{xx}}$.

   (ii) Obtain BLUE of β_1 (5,7)

8. Write short notes on any **three** :
   (i) Sign Test
   (ii) Multiple Linear Regression
   (iii) Run Test
   (iv) Confidence Interval for Proportions (4,4,4)

(100)