This question paper contains 16+8 printed pages]

Your Roll No................

5437

B.A. Prog./III J

(A)

MATHEMATICS – Paper III

(Selected Topics in Mathematics)

(NC – Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :— The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

Attempt *six* questions in all. Unit I and Unit II are compulsory and contain four questions. In Unit III choose any of the options and attempt *two* questions from the same. Marks are indicated.

P.T.O.

Unit I

1. (*a*) Prove that the union of a finite number of closed sets is closed. What happens if the family consists of infinite number of closed sets ? Justify. 6

(*b*) Define a bounded above set, a bounded below set, supremum of a bounded above set and infimum of a bounded below set. If

$$S = \left\{-1, \frac{1}{2}, -\frac{1}{3}, \ldots\ldots, (-1)^n \frac{1}{n}, \ldots\ldots\right\}$$

Find Sups and Infs. 6

Or

(*a*) Define limit point of a set. State Bolzano-Weierstrass'theorem and find the limit points of the set R ~ Q of irrationals. 6

(*b*) Prove that a function uniformly continuous on an interval is continuous on that interval. What is about the converse ? Justify. 6

2. (*a*) Prove :

(*i*) $\lim_{n \to \infty} n^{1/n} = 1$

(*ii*) $\lim_{n \to \infty} \frac{1}{n}\left[1 + 2^{1/2} + 3^{1/3} + \ldots\ldots + n^{1/n}\right] = 1.$ 7

(*b*) Test for convergence any *two* of the following series :

(*i*) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n}$

(*ii*) $\sum_{n=1}^{\infty} \frac{n^{n^2}}{(n+1)^{n^2}}$

(*iii*) $\frac{1}{3} + \frac{1.2}{3.5} + \frac{1.2.3.}{3.5.7.} + \frac{1.2.3.4}{3.5.7.9} + \ldots\ldots$ 6

Or

(a) Define a monotonically increasing sequence and prove that a monotonically increasing sequence that is bounded above converges. 7

(b) Define a conditionally convergent series. Test the convergence and absolute convergence of the series :

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots\ldots \qquad 6$$

3. (a) State Fundamental Theorem of calculus and show that the function f defined by

$$f(x)=x[x],\ \forall\ x\in[0,3]$$

([x] denotes the greatest integer function)

is Riemann integrable on [0, 3]. Also evaluate

$$\int_0^3 f(x)\,dx. \qquad 7$$

(b) Discuss the convergence of :

(i) $\int_a^\infty \frac{dx}{x^n}, a > 0$

(ii) $\int_0^{\pi/2} \frac{\sin x}{x^n} dx.$ 6

Or

(a) Define Beta function, Gamma function and give relation between Beta and Gamma functions. Prove that :

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}.$$ 7

(b) (i) Prove that the series

$$\frac{\sin x}{\sqrt{1}} + \frac{\cos 2x}{\sqrt{2}} + \frac{\sin 3x}{\sqrt{3}} + \frac{\cos 4x}{\sqrt{4}} + \dots\dots$$

is not a Fourier series.

(ii) Find the radius of convergence of the power series :

$$\sum_{n=0}^{\infty} \frac{x^n}{\lfloor n}. \qquad 6$$

Unit II

Computer Programming

4. (a) Calculate the final value of b in the following sequence of statements : 6

```
float b;
int i;
b=2.54;
b=(b+0.05)*10;
i=b;
b=i;
b=b/10.0;
```

(b) Write a program to check whether a square matrix is symmetric or not. 7

Or

(a) Write C expressions to the following :

(i) $\frac{nv^2}{2.5} + \frac{gh}{4d}$

(ii) $\frac{ax+b}{c} + \frac{2f}{dy+e} - \frac{a}{bd} + c.$ 6

(b) Given a five digit integer. Write a program to reverse the number and print it. 7

Unit III (1)

Numerical Analysis

(*Note* :— Use of scientific calculator is allowed.)

5\. (a) Find the root of the equation

$$e^x = 2x + 1.$$

Correct to 4 places of decimals, using Newton-Raphson method. 6

(b) Solve the following system of equations, by Gauss-Seidel iteration method : 6

$$50x_1 + 2x_2 - 3x_3 + 6x_4 = 190$$

$$x_1 - 3x_2 + 2x_3 - 55x_4 = 60$$

$$3x_1 + 65x_2 + 2x_3 + x_4 = 80$$

$$-x_1 + x_2 + 33x_3 + 3x_4 = 60$$

Or

(a) Find the root of the equation

$$\sin x - \cosh x + 1 = 0.$$

Correct to 4 decimal places, using Regula-falsi method. The root lies between 1 and 2. 6

(*b*) Solve the following system of equation by Gauss-elimination method : 6

$$x + y + 2z = 4$$

$$3x + y - 3z = -4$$

$$2x - 3y - 5z = -5$$

6. (*a*) Obtain the cubic spline approximation for the function $y = f(x)$ from the following data, given that : 6

$$y_0'' = y_3'' = 0$$

x	:	–1	0	1	2
y	:	–1	1	3	35

(*b*) Evaluate

$$\int_0^1 \frac{dx}{\sqrt{x^3 + 1}},$$

using Simpson's one-third rule with 10 sub-intervals. 6

P.T.O.

Or

(*a*) Use Newton's divided difference formula to find $f(x)$ from the following data : 6

x :	0	2	3	4	6	7
$f(x)$:	0	8	0	–72	0	1008

(*b*) Evaluate :

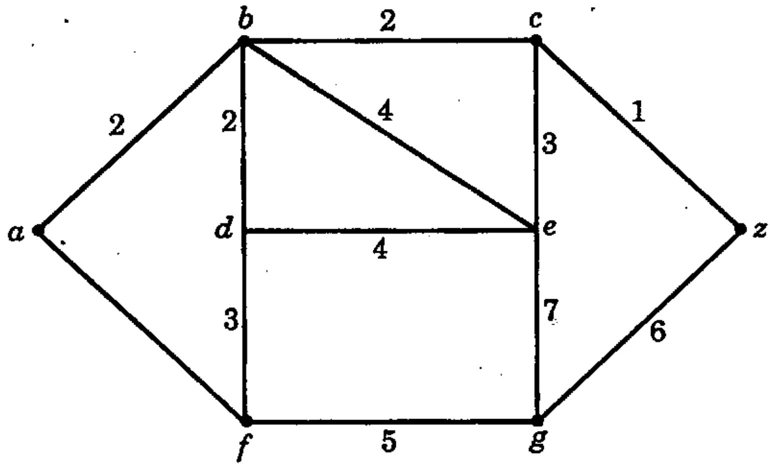
$$I = \int_2^3 \frac{dx}{1+x},$$

using Gauss-Legendre integration method with $n = 4$. 6

Unit III (2)

Discrete Mathematics

5. (*a*) Find the length of a shortest path from vertex a to vertex z in the following connected weighted graph : 6



(*b*) Show that in a graph with n vertices, if there is a path from vertex v_1 to vertex v_2, then there is a path of no more than $n - 1$ edges from v_1 to v_2. 6

Or

(*a*) What is a weighted graph ? Define length of path in a weighted graph. 6

(*b*) For any connected planar graph, prove that :

$$v - e + r = 2.$$

where v, e and r are the number of vertices, edges and regions of the graph, respectively. 6

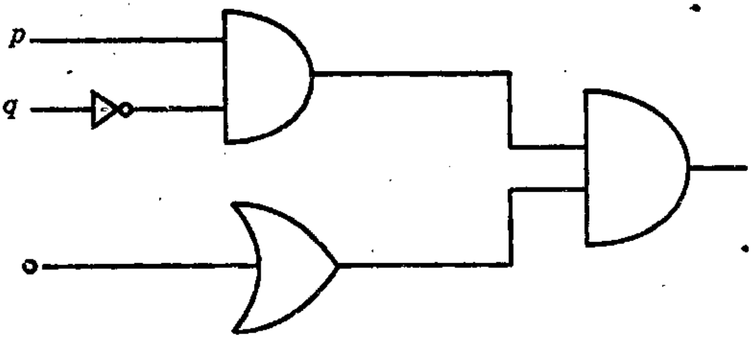
6. (*a*) Let

$$E(x_1, x_2, x_3) = (x_1 \wedge x_2) \vee (x_1 \wedge x_3) \vee (\bar{x}_2 \wedge x_3)$$

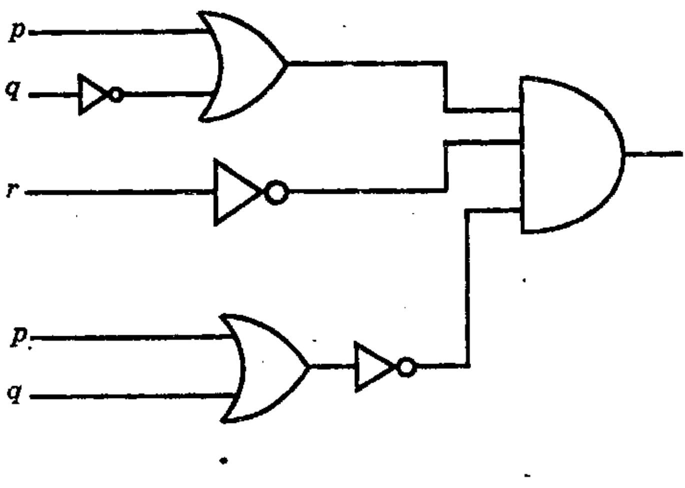
be a Boolean expression over the two-valued Boolean algebra. Write $E(x_1, x_2, x_3)$ in both conjunctive and disjunctive normal forms. 6

(*b*) Write Boolean expression corresponding to the following circuits : 6

(*i*)



(*ii*)



Or

(*a*) Simplify the circuit represented by

$$f = \left(a \wedge \bar{c} \wedge \bar{d}\right) \vee \left(a \wedge \bar{b} \wedge d\right) \vee \left(a \wedge c \wedge \bar{d}\right). \qquad 6$$

(*b*) Write the truth tables for $p \rightarrow q$ and $p \leftrightarrow q$ and show that :

$$p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p). \qquad 6$$

Unit III (3)

Mathematical Statistics

5. (*a*) Prove that the standard deviation is less than the root mean square deviation measured from any other value. 6

(*b*) If A and B are two events and the probability $P(B) \neq 1$, prove that :

$$P\left(A/\bar{B}\right) = \frac{P(A) - P(A \cap B)}{1 - P(B)}$$

where $\bar{B}$ denotes the event complementary to B and hence deduce $P(A \cap B) \geq P(A) + P(B) - 1$. Also show that P(A) > or < P(A/B) according as $P(A/\bar{B})$ > or < P(A). 6

Or

(*a*) Show

$$E(aX + bY) = aE(X) + bE(Y)$$

where X and Y are two stochastic variates, *a* and *b* are constants. 6

(*b*) For a normal distribution, show

$$\beta_1 = 0, \ \beta_2 = 3.$$ 6

6. (*a*) If X and Y are two correlated variables with the same standard deviation and the correlation

coefficient V, show that the correlation coefficient between x and $x + y$ is

$$\sqrt{\frac{1+V}{2}}.$$ 6

(*b*) Find the most probable number of successes in a series of n independent trials, the probability of success in each trial being 'p'. 6

Or

(*a*) Find the first four moments of the Poisson distribution about origin. 6

(*b*) Show that the arithmetic mean of the regression coefficient is greater than the correlation coefficient. 6