This question paper contains 15 printed pages]

Your Roll No. ....

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B.A. Prog./III J

(R)

MATHEMATICS—Paper III

(Selected Topics in Mathematics)

(NC—Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :—The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

P.T.O.

Attempt *All* questions selecting *two* parts from each question. Unit I and Unit II are compulsory and contain *four* questions. In Unit III, choose any of the options and attempt *two* questions from the same.

Marks are indicated.

Unit I

1. (*a*) Define an open set. Which of the following sets are open ? Give arguments in support of your answer : 6

   (*i*) The set Q of rational numbers;

   (*ii*) The interval [0, 2[.

   (*b*) Define a closed set. Prove that the union of a finite number of closed sets is a closed set. What about the union of an infinite family of closed sets ? Justify your answer by means of an example. 6

   (*c*) Show that the function f defined by : 6

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

   is continuous at $x = 0$ only.

(d) Define uniform continuous function defined on an interval I and prove that the function :

$$f(x) = \sqrt{x}$$

is uniformly continuous on [1, 3]. 6

2. (a) Prove that a necessary condition for the convergence of a series :

$$\sum_{n=1}^{\infty} u_n$$

is that $\lim_{n \to \infty} u_n = 0$. Is this condition sufficient also for the convergence ? Justify with a counter example. 7

(b) Show that the sequence $< a_n >$ defined by :

$$a_1 = \sqrt{2}, \; a_{n+1} = \sqrt{2a_n}, \; n \geq 1$$

is monotonic and converges to 2. 7

(c) State Raabe's test for the convergence of positive term series and hence check the convergence of the following series : 7

$$\sum_{n=1}^{\infty} u_n, \; u_n = \frac{1.3....(2n-1)}{2.4.....2n} \frac{x^{2n+1}}{2n+1}.$$

(*d*) Define absolute convergence of a series and prove that an absolutely convergent series is always convergent. What about the converse ? Justify with an example. 7

3. (*a*) Test for pointwise and uniform convergence for the sequence of functions $\{f_n(x)\}$ defined by : $6\frac{1}{2}$

$$f_n(x) = \frac{\log(1+n^4x^2)}{2n^2} \quad 0 \le x \le 1.$$

State clearly the results you are using.

(*b*) Using Beta, Gamma relation, show that : $6\frac{1}{2}$

$$\int_0^\infty \frac{dt}{\sqrt{t}(1+t)} = \pi.$$

(*c*) Test for convergence of the integral : $6\frac{1}{2}$

$$\int_0^\infty e^{-\lambda x} x^{a-1}\, dx \quad (\lambda > 0, a > 0).$$

Unit II

(Computer Programming)

4. (*a*) (*i*) Pick the incorrect decimal integer constants from the following list. For the incorrect decimal

integer constants, explain why they are incorrect : 3

(1) +−785

(2) 1/4

(3) 2A45

(ii) Given an integer, write a program to reverse and print it. For example, if the given number is 12386, the number printed should be 68321. 3

(b) (i) State which values of the control variable *x* are printed by each of the following for statements : 3

(1)
```
for (x = 2; x <= 13; x += 2)
printf("%d \n", x);
```

(2)
```
for (x = 3; x <= 15; x += 3)
printf("%d \n", x);
```

(3)
```
for (x = 12; x >= 2; x -= 3)
printf("%d \n", x);
```

(ii) The trace is defined as the sum of the diagonal elements of a matrix. Write a function to find the trace of a matrix. 3

(c) (i) What is an array ? Give examples. Using examples, explain, how are arrays declared ? 3

(ii) Given the four sides of a rectangle, write a program to find out whether its area is greater than its perimeter. 3

Unit III (1)

(Numerical Analysis)

(Use of scientific calculator is allowed)

5. (a) Describe bisection method to find a root of an equation : 6

$$f(x) = 0.$$

(b) Using the Gauss Elimination method, solve the system of equations : 6

$$10x_1 - x_2 + 2x_3 = 4$$

$$x_1 + 10x_2 - x_3 = 3$$

$$2x_1 + 3x_2 + 20x_3 = 7.$$

(c) Use the Gauss-Jacobi iteration method to solve the system of equations : 6

$$2x_1 - x_2 + 0x_3 = 7$$

$$-x_1 + 2x_2 - x_3 = 1$$

$$0x_1 - x_2 + 2x_3 = 1$$

Perform two iterations only.

6. (a) Fit a cubic, using the Lagrangian interpolating polynomial, through the points (3.2, 22.0), (2.7, 17.8), (1.0, 14.2) and (4.8, 38.3), where each of the four pairs give the value of x and the corresponding value of $f(x)$. Use this cubic to find the interpolated value for $x = 3.0$. 6

(b) Given $(n + 1)$ distinct points $x_0, x_1, x_2, \ldots, x_n$ such that the value of function 'f' at these points i.e. $f(x_0)$, $f(x_1), \ldots, f(x_n)$ are known, then derive the divided difference interpolating polynomial of degree n. 6

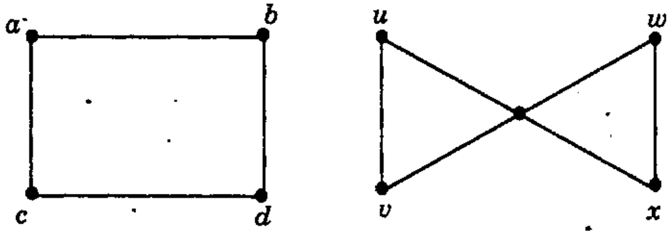
(c) Use Simpson's 1/3 rule to evaluate the integral of e^{-x^2} over the interval 0.2 to 1.5, using 2, 4, 6, ...subdivisions until the answer is correct to two decimal places of accuracy. 6

Unit III (2)

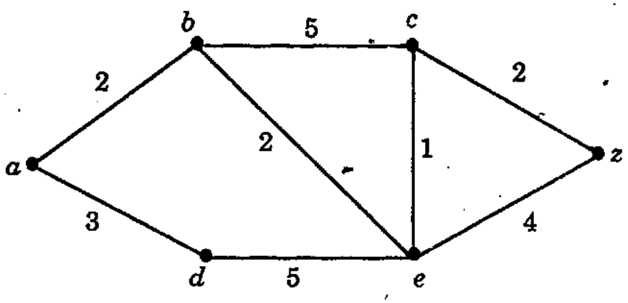
(Discrete Mathematics)

5. (*a*) Suppose that a connected planar simple graph has 20 vertices each of degree 3. Into how many regions does a representation of this planar graph split the plane ? 6

(*b*) Show that the graphs G and H are isomorphic. 6



(*c*) Find the length of a shortest path between a and z in the weighted graph. 6



6. (*a*) Write the following Boolean expression : 6

$$E(x_1, x_2, x_3) = (x_1 \wedge x_2 \wedge \bar{x}_3) \vee (x_1 \wedge \bar{x}_2 \wedge x_4) \wedge (x_2 \wedge \bar{x}_3 \wedge \bar{x}_4)$$

over the two valued Boolean algebra in conjunctive normal form.

(*b*) Draw the switching circuit corresponding to the Boolean expression : 6

$$a \vee (b \wedge d) \vee (\bar{c} \wedge d)$$

(*c*) Show that the following statement is a tautology : 6

$$(A \rightarrow B) \rightarrow [(\bar{A} \rightarrow B) \rightarrow B],$$

where $\bar{A}$ denotes the negation of A.

Unit III (3)

(Mathematical Statistics)

5. (*a*) For any distribution show that the mean deviation is least when measured from the median. 6

(*b*) Suppose X_1, X_2 and X_3 are uncorrelated random variables each having same standard deviation. Obtain the correlation coefficient between : 6

$$X_1 + X_2 \text{ and } X_2 + X_3.$$

(*c*) Let X and Y are two random variables with variances σ_{X^2} and σ_{Y^2} and suppose *r* is the correlation coefficient between them. Find the angle between the two regression lines. 6

6. (*a*) In a bolt factory, three machines A, B, C are manufacturing 25%, 35% and 40% of bolts. In a day's production, it is found that the three machines have manufactured 5%, 4% and 2% defective bolts. A bolt is chosen at random from the product. What is the probability that it is defective ? Knowing that it is defective, what is the probability that it was produced by A ? 3+3=6

(*b*) Using Poisson distribution, find the probability that at most 5 defective fuses will be found in a box of 200 fuses if experience shows that 2% of such fuses are defective. 6

(*c*) A coin is tossed 100 times. Using normal approximation to Binomial distribution, find the probability of getting : 3+3=6

(*i*) Exactly 50 heads;

(*ii*) More than 60 heads.

Unit III (4)

(Mechanics)

5. (*a*) Show that any force system in the fundamental plane may be reduced either to a single force or to a couple. 6

(*b*) Find the mass centre of a wire bent into the form of an isosceles right angled triangle. 6

(*c*) A square frame, consisting of four equal uniform rods of length '2*a*' rigidly jointed together, hangs at rest in a vertical plane on two smooth pegs P, Q at the same level. If PQ = C and pegs are not both in contact with the same rod, show that there are three positions of equilibrium, provided $a < \frac{C}{\sqrt{2}}$, of these show that the only unstable one is the symmetrical one. 6

6. (*a*) A particle moves in a plane $r = ae^{\theta \cot \alpha}$, which is the equiangular spiral, and of the radius vector to the particle has constant angular velocity, show that the resultant acceleration of the particle makes an angle 2α with the radius vector and is of magnitude $\frac{v^2}{r}$, where v is the speed of the particle. 6

(*b*) A gun is fired from a moving platform, and the ranges of the shot are observed to be R and S, when the platform is moving backward and forward respectively, with velocity v. Prove that the elevation of the gun is : 6

$$\tan^{-1}\left[\frac{g(R-S)^2}{4v^2(R+S)}\right].$$

(c) If the central orbit be an ellipse, the focus being the centre of the force, find the law of force, the velocity on the path and the period. Prove with notations : 6

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) \text{ and } h^2 = \mu a(1 - e^2).$$

Unit III (5)

(Theory of Games)

5. (a) What is meant by a feasible solution of a linear programming problem ? Write down the following L.P.P. in standard form : 6

Maximize : $Z = 3x_1 + 2x_2 + 5x_3$

Subject to :

$$2x_1 - 3x_2 \le 3$$

$$x_1 + 2x_2 + 3x_3 \ge 5$$

$$3x_1 + 2x_3 \le 2$$

$$x_1, x_2, x_3 \ge 0.$$

(*b*) Use Charne's penalty or two phase method to solve : 6

Maximize : $Z = 2x_1 + x_2 - x_3$

Subject to :

$$4x_1 + 6x_2 + 3x_3 \leq 8$$

$$3x_1 - 6x_2 - 4x_3 \leq 1$$

$$2x_1 + 3x_2 - 5x_3 \geq 4$$

$$x_1, x_2, x_3 \geq 0.$$

(*c*) Apply principle of duality to solve : 6

Minimize : $Z = -2x_1 + 3x_2 + 4x_3$

Subject to :

$$-2x_1 + x_2 \geq 3$$

$$-x_1 + 3x_2 + x_3 \geq -1$$

$$x_1, x_2, x_3 \geq 0.$$

6. (*a*) Explain the Max-Min and Min-Max principle used in game theory. Determine the saddle point of a game whose payoff matrix is : 6

Player B

		B_1	B_2
Player A	A_1	–1	6
	A_2	2	4
	A_3	–2	–6

(*b*) Use dominance principle to reduce the following game to 2 × 2 games and hence solve them : 6

Player C

		C_1	C_2	C_3
Player R	R_1	2	0	3
	R_2	3	–1	1
	R_3	5	2	–1

(*c*) Reduce the following game to an L.P.P. and hence solve : 6

$$\begin{bmatrix} 2 & -2 & 3 \\ -3 & 5 & -1 \end{bmatrix}.$$