Your Roll No. ....

4493-A   **AS**

B.A./III

MATHEMATICS–Paper-III (*ii*)

(Real Analysis and Statistics)

Time : 3 *Hours*   *Maximum Marks* : 100

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :   Attempt Part A (Real Analysis) and Part B (Statistics) in separate answer-sheets.

All questions carry equal marks.

Attempt any *two* parts from each Section.

Part A (Real Analysis)

Section I

1.   (*a*)   Define limit point of a set. Determine set of limit points of the following sets :

(*i*) Q = the set of all rational numbers,

(*ii*) $T =]a, b[$

(*iii*) $T = \left\{ \dfrac{1}{n} : n = 1, 2, 3, \ldots \ldots \right\}$

(*b*)   Define an open set. Prove that union of an arbitrary family of open sets not an open set.

(c) State and prove Intermediate Value Theorem.

(d) Define a Uniformly Continuous function. Prove that a uniformly continuous function is continuous.

Section II

2. (a) Show that $\lim\limits_{n \to \infty} n^{1/n} = 1$.

(b) Prove that a monotonic sequence is convergent if and only if it is bounded.

(c) Discuss the convergence of the series :

(i) $\sum\limits_{n=1}^{\infty} \dfrac{x^n}{n^2}$ $(x > 0)$

(ii) $\dfrac{1}{3} + \dfrac{2}{5} + \dfrac{3}{7} + \ldots\ldots$

(d) Discuss the convergence, conditional convergence and absolute convergence of the series :

$$\sum\limits_{n=2}^{\infty} (-1)^n \dfrac{1}{\log n}$$

Section III

3. (a) Show that for $0 < u < v$, there exist c, $u < c < v$

$$\dfrac{\tan^{-1} v - \tan^{-1} u}{v - u} = \dfrac{1}{1 + c^2}$$

(b) State and prove Taylor's Theorem with Lagrange's form of remainder.

(c) Prove that :

$$1+x+\frac{x^2}{2!} < e^x < 1+x+\frac{x^2}{2!}\ e^x, \text{ for } x > 0.$$

(d) Evaluate the limit,

(i) $\lim\limits_{x \to 0} \dfrac{2^x - 1}{\sin x}$

(ii) $\lim\limits_{x \to 0} x^{\tan x}$

Part B (Statistics)

Section IV

4. (a) A manufacturer of bolts knows that 5% of his product is defective. If he sells bolts in boxes of 100 and guarantees that not more than 10 bolts will be defective, what is the probability that a box will fail to meet the guaranteed quality?

(b) Find arithmetic mean, standard deviation, harmonic mean, mode and median of the distribution given by

$$f(x) = 6\,(x - x^2), \quad 0 \le x \le 1.$$

(c) In a partially destroyed Laboratory records of the correlation analysis of data, the following results only are legible. Var $X = 9$, Regression lines :

$8x - 10y + 66 = 0$ and $40x - 18y = 214$.

Find the mean value of X and Y and the correlation coefficient between X and Y.

(d) The distribution of x is such that :

$$P(X = k) = q^{K-1} p, K = 1, 2, \ldots \ldots \text{ and } p + q = 1$$

find $E[X]$ and Var $[X]$.

Section V

5. (a) For a Binomial Distribution, show that

$$u_{r+1} = pq \left\{ nr\mu_{r-1} + \frac{d\mu_r}{dp} \right\}$$

and deduce the values of μ_2, μ_3 and μ_4.

(b) Show that, if a variate x takes values 0, 1, 2, ..., n with frequencies proportional to ${}^nC_0$, ${}^nC_1$, ..., ${}^nC_n$, respectively, then the mean of distribution is $\frac{n}{2}$ and variance is $\frac{n}{4}$.

(c) Show that correlation coefficient cannot numerically exceed unity and is independent of change of origin and scale.

(d) Assume a Normal Distribution with $N = 1,000$, $\mu = 80$, $\sigma = 15$.

(i) How many observations may be expected to lie between 65 and 110?

(ii) Find the value of variate beyond which 10% of items would lie.

Section VI

6. (a) Two types of batteries A and B are tested for their length of life and following results were obtained.

Size of sample		Mean	Variance
A	10	500 hrs	100
B	10	560 hrs	121

Is there a significant difference in the two means?

(b) An experiment succeeds twice as often as it fails. Find the chance that in the next six trials there will be at least four successes.

(c) Fit a straight line to the following data :

X :	0	5	10	15	20	25
Y :	12	15	17	22	24	30

(d) The means of two single large samples of 1000 and 2000 members are 67.5 cm and 68.0 cm respectively. Can the samples be regarded as drawn from the same population of standard deviation 2.5 cm?