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B.A. Prog./III   AS

APPLICATION COURSE— MATHEMATICS FOR

SOCIAL SCIENCES

(Admissions of 2004 & onwards)

Time : 2 Hours   *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. 1 is compulsory and carries 15 marks.

Attempt *four* more questions selecting

at least *one* question from each Section.

Each question carries 10 marks.

1.   (*i*)   Given

$$A = \begin{bmatrix} 2 & 8 \\ 3 & 0 \\ 5 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 3 & 8 \end{bmatrix}.$$

Calculate AB. Can you calculate BA ? Explain your answer.

P.T.O.

(*ii*) Show that :

$$\lim_{x \to 4} \frac{x^2 - 9x + 20}{x^2 - 3x - 4} = -\frac{1}{5}.$$

(*iii*) If $y = x^x$, find $\frac{dy}{dx}$.

(*iv*) If $z = \log(x^2 + y^2)$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(*v*) Solve the following differential equation :

$$xy\ dx + (1 + x^2)\ dy = 0. \qquad\qquad 5\times3$$

Section I

2. (*i*) Show that the function $f(x) = x^3 - 6x^2 + 9x - 8$ has a maximum value at $x = 1$ and a minimum value at $x = 3$.   5

(*ii*) Sketch the graph of the parabola $x^2 = 12y$.   5

3. (*i*) Find $\int x^2 \log x\ dx$.   5

(*ii*) Examine the continuity of the function

$$f(x) = \begin{cases} 2 - x, & \text{if } x \le 1 \\ 2 - x, & \text{if } x > 1 \end{cases}$$

at $x = 1$.   5

Section II

4. (i)   Write down the Taylor's series for $\sin x$ and compute $\sin (0.1)$ to three places of decimal.   5

    (ii) Test for convergence the series whose nth term is $\left(\sqrt{n+1} - \sqrt{n}\right)$.   5

5. (i)   For what value of λ are the vectors $\vec{a} = \lambda\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 4\hat{i} - 9\hat{j} + 2\hat{k}$. are perpendicular to each other ?   5

    (ii) Solve the following differential equation :

    $$y(1 + x)\, dx + x\,(1 + y)\, dy = 0.$$ 5

Section III

6. (i)   Find the rank of the matrix :

    $$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}.$$ 5

    (ii) Solve the following system of equations :

    $$x - 3y + z = -1,$$

    $$2x + y - 4z = -1,$$

    $$6x - 7y + 8z = 7.$$ 5

7. (*i*)   The production function of a firm is given by :

$$Q = 8LK - L^2 - K^2, L > 0, K > 0.$$

Find the marginal productivities of Labour and Capital. Also show that :

$$L\frac{\partial Q}{\partial L} + K \cdot \frac{\partial Q}{\partial K} = 2Q.$$ 5

(*ii*)   Use graphical method to solve the following linear programming problem :   5

Maximize : $Z = 10x + 6y$

Subject to the constraints :

$$3x + y \leq 12,$$

$$2x + 5y \leq 34,$$

$$x, y > 0.$$