This question paper contains 16 printed pages.]

Your Roll No. ....

885

B.A. Prog. / III **A**

(L)

MATHEMATICS – Paper III

(Selected Topics in Mathematics)

(Admissions of 2004 and onwards)

Time : 3 Hours **Maximum Marks : 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note : The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

Attempt **six** questions in **all. Unit I** and **Unit II** are compulsory and contain **four** questions. In **Unit III** choose any of the options and attempt **two** questions from the same. Marks are indicated against each question.

Unit – I

1. (a) Define a bounded above set, a bounded below set, supremum of a bounded above set and infimum of a bounded below set. If

$$S = \left\{-1, \frac{1}{2}, \frac{-1}{3}, \frac{1}{4}, \ldots, (-1)^n \cdot \frac{1}{n}, \ldots\right\}$$

Find Sup S and Inf S. **6**

(b) Prove that the intersection of a finite number of open sets is open. What happens if the family consists of infinite number of open sets ? Justify your answer. **6**

OR

(a) Define limit point of a set. State Bolzano Weierstrass Theorem and show by an example that condition of boundedness in the theorem can not be relaxed. **6**

(b) Prove that a function continuous on a closed interval [a, b] is bounded on [a, b]. **6**

2. (a) Prove that every convergent sequence is bounded. Is every bounded sequence convergent ? Justify. **7**

(b) Test for convergence any **two** of the following series :

(i) $\sum_{n=1}^{\infty} \left(\sqrt{n+1} - \sqrt{n}\right)$

(ii) $\sum_{n=2}^{\infty} \frac{1}{(\log n)^n}$

(iii) $\frac{1}{5} + \frac{\lfloor 2}{5^2} + \frac{\lfloor 3}{5^3} + \dots + \frac{\lfloor n}{5^n} + \dots$ **6**

OR

(a) Prove that $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$ exists and lies between 2 and 3. **6**

(b) State and prove Cauchy's n^{th} root test for the convergence of an infinite series of positive terms. **7**

3. (a) Let f be a continuous function defined on a closed and bounded interval [a, b]. Prove that f is Riemann integrable on [a, b]. **7**

(b) Show that the integral

$$\int_0^1 x^{m-1} (1-x)^{n-1}\, dx$$

converges if and only if m > 0, n > 0. **6**

OR

(a) Define Gamma function and prove that :

$$\sqrt{\pi}\,\Gamma(2m) = 2^{2m-1}\,\Gamma(m)\,\Gamma\left(m + \frac{1}{2}\right), \; m > 0. \qquad 7$$

(b) Examine the pointwise and uniform convergence of $\langle f_n(x) \rangle$ where

$$f_n(x) = \frac{\sin nx}{\sqrt{n}}, \; 0 \le x \le 2\pi \qquad 6$$

Unit – II

Computer Programming

4. (a) Calculate the final value of b in the following sequence of statements.

```
float b ;
int a ;
b = 2.56 ;
b = (b + 0.05) * 10 ;
i = b ;
b = i ;
b = b/10.0 ;
```

6

(b) Write a program to multiply two matrices of order 2×3 and 3×2. 7

OR

(a) Write C expressions corresponding to the following :

(i) $$\frac{xy + \frac{z}{np + j} + x}{y} + z$$

(ii) $x^5 + 10x^4 + 8x^3 + 4x + 2$ 6

(b) Give a set of integers, count the number of negative, zero and positive integers. 7

Unit – III (1)

Numerical Analysis

Note : Use of scientific calculator is allowed.

(a) Find the iterative method based on the Newton-Raphson method for finding $\sqrt{N}, \frac{1}{N}$, where N is a positive real number. Apply the methods to $N = 18$ to obtain the results correct to two decimal places. 6

(b) Determine the convergence factor for the Jacobi and Gauss-Seidel methods for the system.

$$\begin{bmatrix} 4 & 0 & 2 \\ 0 & 5 & 2 \\ 5 & 4 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix}$$

OR

(a) To solve the system of equations

$$4x + y + 2z = 4$$

$$3x + 5y + z = 7$$

$$x + y + 3z =$$

Set up the Gauss-Seidel iterative scheme and iterate three times starting with the initial vector $X^{\circ} = 0$. Compare with the exact solution.

(b) Find the interval in which the smallest positive root of the equation

$x^3 - x - 4 = 0$ lies

Determine the root correct to two decimal places, using the bisection method. 6

(a) Let $f(x) = \log(1 + x)$, $x_0 = 1$ and $x_1 = 1.1$. Use linear interpolation to calculate an approximate value of f(1.04) and obtain a bound on the truncation error. **6**

(b) Evaluate the integral $\int_{-1}^{1} e^{-x^2} \cos x \, dx$, using Gauss-Legendre 3-point formula. **6**

OR

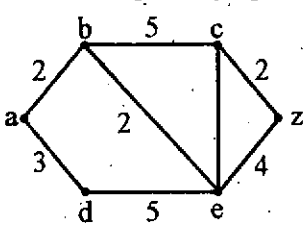
(a) Find the divided differences of $f(x) = x^3 - x^2 + 3x + 8$ for the arguments 0, 1, 4, 5. **6**

(b) Compute $\int_{0}^{\pi/2} \sin dx$, using Simpson's three eighth's rule of integration. **6**

Unit – III (2)

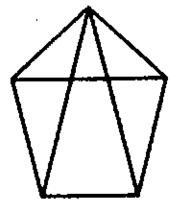
Discrete Mathematics

(a) Find the length of a shortest path between a and z in the weighted graph.



6

(b) Find the planar graph of the following :



6

OR

(a) Define the following :

(i) Undirected graphs

(ii) Sub graph

(iii) Walk, Path and Circuit 6

(b) Show that any simple, connected graph with 31 edges and 12 vertices is not planar. 6

6. (a) Define the following :

(i) an AND gate

(ii) an OR gate

(iii) a NOT gate

(b) Write the logic table and output of the following circuit :

(i) 

(ii) 

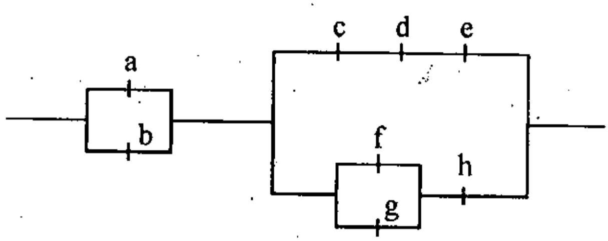
6

OR

(a) Define conjunctive normal form and disjunctive normal form. Find the disjunctive normal form of the function whose conjuctive normal form is

$f = (x \vee y \vee z) \wedge (x \vee y \vee \overline{z}) \wedge (x \vee \overline{y} \vee z) \wedge (x \vee \overline{y} \vee \overline{z}) \wedge (\overline{x} \vee y \vee z)$ 6

(b) Find the function f that represents the circuit



and then find the circuit which would be open (closed) when the above circuit is closed (open). 6

(b) Draw the circuits represented by the Boolean functions

(i) $f = a \wedge (b \vee c)$

(ii) $f = a \wedge [(b \vee \bar{d}) \vee (\bar{c} \wedge (a \vee d \vee \bar{c}))] \wedge b$ **6**

Unit – III (3)

Mathematical Statistics

5. (a) Find the mean deviation from the mean and the standard deviation of the Arithmetic Progression a, a + d, ……., a + 2nd and prove that the latter is greater than the former. **6**

(b) Find the mean, variance, moment generating function of the Poisson distribution. **6**

OR

(a) Show that the mean deviation from the mean of the normal distribution is about $\frac{4}{e}$ of its standard deviation. **6**

(b) Fit a straight line to the data :

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

6

6. (a) If X_1, X_2, X_3 are uncorrelated variables each having the same standard deviation, obtain the correlation coefficient between $u = x_1 + x_2$ and $v = x_2 + x_3$. 6

(b) State and prove the theorem of total probability. 6

OR

(a) Show that E(XY) = E(X) E(Y) where X and Y are two independent random variables. 6

(b) Define moment generating function of a random variable and prove its properties. 6

Unit – III (4)

Mechanics

5. (a) Forces P_1, P_2, P_3, P_4, P_5, P_6 act along the sides of a regular hexagon taken in order. Show that they will be in equilibrium if

$$P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = 0$$

and $P_1 - P_4 = P_3 - P_6 = P_5 - P_2$. 6

(b) Find the C.G. of a solid uniform hemisphere of radius 'a'. 6

OR

(a) A and B are two fixed points in a horizontal line at a distance 'c' apart. Two fine light strings AC and BC of lengths 'b' and 'a' respectively support a mass at C. Show that the tensions of the strings are in the ratio.

$b(a^2 + c^2 - b^2) : a(b^2 + c^2 - a^2)$. 6

(b) A regular hexagon ABCDEF consists of six equal rods which are each of weight W and are freely jointed together. The hexagon rests in a vertical plane and AB is in contact with a horizontal table. If C and F be connected by a light string, prove that its tension is $W\sqrt{3}$. 6

6. (a) A body travels a distances 's' in 't' seconds. It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration f and in the second part with constant retardation r. Show that

$$t = \sqrt{2s\left(\frac{1}{f} + \frac{1}{r}\right)}$$ 6

(b) If 'l_1' be the length of an imperfectly adjusted seconds pendulum which gains 'n' seconds in one hour and 'l_2' the length of one which loses 'n' seconds in one hour, at the same place, show that true length of seconds pendulum is

$$\frac{4l_1 l_2}{l_1 + l_2 + 2\sqrt{l_1 l_2}}$$ 6

OR

(a) If v_1 and v_2 be the velocities at the ends of a focal chord of a projectile's path and 'u', the horizontal component of velocity, show that

$$\frac{1}{v_1^2} + \frac{1}{v_2^2} = \frac{1}{u^2}$$ 6

(b) Assuming that the eccentricity 'e' of a planet's orbit is a small fraction, show that the ratio of the time taken by the planet to travel over the halves of its orbit separated by the minor axis is nearly

$$1 + \frac{4e}{\pi}$$ 6

Unit – III (5)

Theory of Games

5. (a) Solve graphically the following linear programming problem :

Maximize $Z = 2.5x_1 + x_2$

Subject to $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0$ **6**

(b) Write down the dual of the following linear programming problem :

Maximize $Z = 6x_1 + 4x_2 + x_3 + 7x_4 + 5x_5$

Subject to. $3x_1 + 7x_2 + 8x_3 + 5x_4 + x_5 = 2$

$2x_1 + x_2 + 3x_3 + 9x_5 = 6$

$x_1, x_2, x_3, x_4 \geq 0, x_5$ is unrestricted **6**

OR

(a) Solve the following linear programming problem :

Maximize $Z = 107x_1 + x_2 + 2x_3$

Subject to $14x_1 + x_2 - 6x_3 + 3x_4 = 7$

$16x_1 + x_2 - 6x_3 \leq 5$

$3x_1 - x_2 - x_3 \leq 0$

$x_1, x_2, x_3, x_4 \geq 0$ **6**

(b) Using Charne's Big M method solve the following linear programming problem :

Minimize $Z = 2x_1 + x_2$

Subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$ **6**

(a) Solve graphically the game whose pay-off matrix is

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \\ 2 & 2 \end{bmatrix}$$

6

(b) Find the range of values of p & q which will render (2, 2) a saddle point of the game

$$\begin{bmatrix} 4 & p & 2 \\ q & 5 & 7 \\ 10 & 3 & 9 \end{bmatrix}$$

OR

(a) Use dominance to solve the following game :

$$\begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \\ 5 & 1 & 6 \end{bmatrix}$$

(b) Transform the matrix game

$$\begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 3 & 2 & -3 \end{bmatrix}$$

into its corresponding primal and dual linear programming problem and solve.
