This question paper contains 16 printed pages.]

Your Roll No. ....

886

B.A. Prog. / III A

(T)

MATHEMATICS – Paper III

(Selected Topics in Mathematics)

(Admissions of 2004 and onwards)

Time : 3 Hours **Maximum Marks : 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note : The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

Attempt **six** questions in all. Unit **I** and Unit **II** are compulsory and contain **four** questions. In Unit **III**, choose any of the options and attempt **two** questions from the same. Marks are indicated.

Unit – I

1. (a) Define an open set of real numbers. Show that a non empty subset of R cannot be open. Is the set of rationals open ? Justify with proof. **6**

   (b) State the properties which make R a complete ordered field. **6**

OR

   (a) Define limit point of a set. State Bolzano Weierstrass Theorem and show by an example that condition of boundedness in the theorem cannot be relaxed. **6**

   (b) Show that the function f defined on R by

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational} \end{cases}$$

   is discontinuous at every point. **6**

2. (a) Test for convergence any **two** of the following series :

   (i) $\frac{1}{3.7} + \frac{1}{4.9} + \frac{1}{5.11} + \frac{1}{6.13} + \ldots$

   (ii) $\sum_{n=1}^{\infty} 2^{-n-(-1)^n}$

   (iii) $\frac{1}{2} + \frac{\lfloor 2}{8} + \frac{\lfloor 3}{32} + \frac{\lfloor 4}{128} + \ldots$ **6**

(b) Prove that every convergent sequence is bounded. Is every bounded sequence convergent ? Justify. 7

OR

(a) Define a monotonically increasing sequence and prove that a monotonically increasing sequence that is bounded above converges. 7

(b) State Leibnitz Test for the convergence of an alternating series. Test the convergence and absolute convergence of the series 6

$$\frac{1}{2}-\frac{1}{4}+\frac{1}{6}-\frac{1}{8}+\dots$$

3. (a) Show that the improper integral

$$\int_{0}^{\infty} e^{-x} x^{n-1} dx$$

Converges if and only if $n > 0$. 6

(b) Let f be a monotonically increasing function defined on a closed and bounded interval [a, b]. Prove that f is Riemann integrable on [a, b]. 7

OR

(a) Show that :

(i) $\Gamma(1) = 1$

(ii) $\beta(m, n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx \; m > 0, n > 0$ **6**

(b) (i) Prove that the series

$$\frac{\sin x}{\sqrt{1}} + \frac{\cos 2x}{\sqrt{2}} + \frac{\sin 3x}{\sqrt{3}} + \frac{\cos 4x}{\sqrt{4}} + \ldots$$

is not a Fourier series.

(ii) Find the radius of convergence of the power series **7**

$$\sum_{n=0}^{\infty} \frac{x^n}{\lfloor n}$$

Unit – II

Computer Programming

4. (a) Evaluate the following expressions : float a = 2.5, b = 2.5 ; **6**

(i) a + 2.5 / b + 4.5

(ii) (a + 2.5) / b + 4.5

(iii) (a + 2.5) / (b + 4.5)

(iv) a / 2.5 / b

(b) Write a program to obtain the transpose of an $m \times n$ matrix. 7

OR

(a) Write C expressions corresponding to the following : 6

(i) $\dfrac{ab}{c + \dfrac{dR}{m} + R} + a$

(ii) $a^3 + b^3 + c^3 - 3ab\,(a + b)$

(b) Write a program to find whether a given point (x, y) lies on the x-axis, y-axis or at origin namely (0, 0). 7

Unit – III (1)

Numerical Analysis

(**Note :** Use of scientific calculator is allowed.)

5. (a) Find the negative root of the equation $x^3 + 2x + 10 = 0$, correct to 3 places. 6

(b) Solve the following system of equation by Gauss elimination method : **6**

$$5x_1 - x_2 = 9$$

$$-x_1 + 5x_2 - x_3 = 4$$

$$-x_2 + 5x_3 = -6$$

OR

(a) Find the root of the equation

$x^x = 100$, correct to 4 places of decimals, using Newton-Raphson method. **6**

(b) Solve the following system of equations, by using Gauss-Jordan elimination method : **6**

$$x_1 + x_2 + x_3 + x_4 = 2$$

$$x_1 + x_2 + 3x_3 - 2x_4 = -6$$

$$2x_1 + 3x_2 - x_3 + 2x_4 = 7$$

$$x_1 + 2x_2 + x_3 - x_4 = -2$$

6. (a) Find the values of y when $x = 3.2$ and when $x = 2.9$ from the following data using Gauss's forward formula : **6**

x	2.0	2.5	3.0	3.5	4.0
y	246.2	409.3	537.2	636.3	715.9

(b) Evaluate the integral $I = \int_0^1 \frac{dx}{1+x^2}$, using 3 point Gauss-Legendre's Quadrature formula. **6**

OR

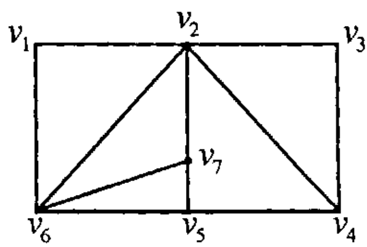
(a) For the equally spaced tabular data for $y = f(x)$, prove that the error in (Newton's forward) linear interpolation does not exceed $\frac{1}{8}$ times the second difference. **6**

(b) Evaluate $\int_1^3 \int_1^2 \frac{dx\,dy}{xy}$, using trapezoidal rule with $h = k = 0.5$. **6**

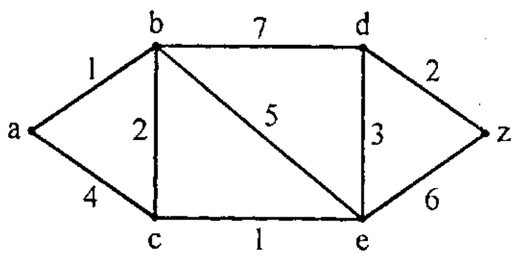
Unit – III (2)

Discrete Mathematics

5\. (a) Find a Hamiltonian cycle in the graph. **6**



(b) Obtain a shortest path from the vertex a to vertex z in the weighted graph. **6**



OR

(a) Show that in any simple, connected, planar graph, $e \leq 3v - 6$. Give an example of a simple, connected and non-planar graph for which $e \leq 3v - 6$ **6**

6. (a) What is a combinatorial circuit ? Find the combinatorial circuit corresponding to the Boolean expression.

$$\left(x_1 \wedge \left(\bar{x}_2 \vee x_3\right)\right) \vee x_2$$

and write the logic table for the circuit obtained. **6**

(b) Prove the equation : **6**

$$(a \vee \bar{b} \vee d) \wedge (a \vee b \vee \bar{c}) \wedge (a \vee c \vee d) = a \vee (b \wedge d) \vee (\bar{c} \wedge d)$$

OR

(a) Write Boolean functions corresponding to the Boolean expressions :

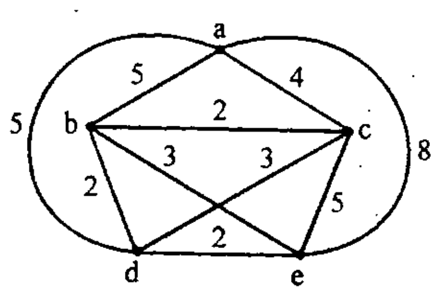
(i) $\left(\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3\right) \vee \left(\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3\right) \vee$ $(x_1 \wedge x_2 \wedge x_3)$

(ii) $\left(x_1 \wedge \bar{x}_2 \wedge x_3\right) \vee \left(\bar{x}_1 \wedge x_2 \wedge \bar{x}_3\right) \vee$ $\left(x_1 \wedge x_2 \wedge \bar{x}_3\right)$

in tabular form specifying values of f corresponding to various possible combinations of values of x_1, x_2 and x_3. 6

(b) Simplify the circuit represented by $f = \left(a \wedge \bar{c} \wedge \bar{d}\right) \vee \left(a \wedge \bar{b} \wedge d\right) \vee \left(a \wedge c \wedge \bar{d}\right)$ 6

(b) Use the nearest neighbour method to determine a Hamiltonian circuit for the following graph, starting from the vertex 'a' : 6



Unit – III (3)

Mathematical Statistics

5. (a) Prove that for any discrete distribution, standard deviation is not less than the mean deviation from mean. **6**

(b) Find the mode of the Poisson distribution. **6**

OR

(a) State and prove the Baye's theorem. **6**

(b) Fit a parabola to the following data taking x as the independent variable : **6**

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

6. (a) Prove that for a normal distribution, the quartile deviation, mean deviation and the standard deviation are approximately in the ratio

10 : 12 : 15 **6**

(b) State and prove the Bon Ferronis Inequality. **6**

OR

(a) Prove that the mean deviation is least when measured from median. 6

(b) Prove the recurrence relation for the binomial distribution

$$\mu_{r+1} = pq\left(n_r \mu_{r-1} + \frac{d\mu r}{dp}\right)$$

where μ_r is the r^{th} moment about the mean. Hence obtain μ_2, μ_3, μ_4. 6

Unit – III (4)

Mechanics

5. (a) Show that the resultant of two forces P and Q inclined at an angle α to reach other, is $(P^2 + Q^2 + 2PQ \cos \alpha)^{\frac{1}{2}}$ making with P an angle $\sin^{-1}\frac{Q \sin \alpha}{\sqrt{[P^2 + Q^2 + 2PQ \cos \alpha]}}$. 6

(b) Find the position of the centroid of the area of the curve $ay^2 = x^3$ between the origin and $x = b$. 6

OR

(a) A uniform rod AB of weight W and length 2a can turn about a smooth hinge at its upper end A, and the lower end B is kept at a horizontal distance 2b from the hinge by a horizontal force applied at B. Prove that the reaction at the hinge is

$$\frac{1}{2} W \sqrt{[(4a^2 - 3b^2) / (a^2 - b^2)]}$$ 6

(b) A uniform ladder of length l has its upper end projecting slightly over a smooth horizontal rail at a height h. If the ladder is about to slip and λ is the angle of friction prove that

$$\tan \lambda = \left\{h\sqrt{l^2 - h^2}\right\} / (l^2 + h^2)$$ 6

6. (a) If a particle is moving according to the law $v^2 = 2(x \sin x + \cos x)$, where x is the distance described, find its acceleration. 6

(b) An elastic string of natural length l is extended by an amount 'x' when it supports a mass 'm' at rest, and is extended by an amount 'y' when it is rotating as a conical pendulum carrying a particle of the same mass. Show that

$$gy = w^2 x(l + y)$$ **6**

OR

(a) Prove that if the time of flight of a bullet over a horizontal range R is T seconds, the inclination of the direction of projection to the horizontal is $\tan^{-1}\left(\frac{gT^2}{2R}\right)$ **6**

(b) A particle describes an orbit with a central acceleration. $\mu u^3 - \lambda u^5$. Being projected from an apse at a distance 'a' with a velocity equal to that from infinity. Show that its path is

$$r = a \cosh \frac{\theta}{n} \text{ where } n^2 + 1 = \frac{2\mu a^2}{\lambda}.$$ **6**

Unit – III (5)

Theory of Games

5. (a) Solve graphically the following linear programming problem :

Maximize $Z = x_1 + x_2$

Subject to $x_1 + x_2 \leq 1$

$-3x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$ **6**

(b) Use Simplex method to solve the linear programming problem

Maximize $Z = 10x_1 + x_2 + 2x_3$

Subject to $x_1 + x_2 - 2x_3 \leq 10$

$4x_1 + x_2 + x_3 \leq 20$

$x_1, x_2, x_3 \geq 0$ **6**

OR

(a) Solve using Charne's Big M technique

Maximize $Z = 3x_1 + 2x_2 + 3x_3$

Subject to $2x_1 + x_2 + x_3 \leq 2$

$3x_1 + 4x_2 + 2x_3 \geq 8$

$x_1, x_2, x_3 \geq 0$ **6**

(b) Write down the dual of the following problem in a form such that the dual variables are non-negative

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

Subject to $2x_1 + 3x_2 + 9x_3 \geq 9$

$x_1 - x_2 + 7x_3 \leq 8$

$2x_1 - 5x_2 - x_3 = 11$

$x_1, x_2 \geq 0$, x_3 is unrestricted. **6**

6. (a) Use dominance to solve the following game : **6**

$$\begin{bmatrix} 8 & 15 & -4 & -2 \\ 19 & 15 & 17 & 16 \\ 5 & 20 & 15 & 5 \end{bmatrix}$$

(b) Use graphical method to solve the rectangular game whose pay-off matrix is **6**

$$\begin{bmatrix} 6 & 2 & 7 \\ 1 & 9 & 3 \end{bmatrix}$$

OR

(a) Solve the game whose pay-off matrix **6**

$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 3 \\ -1 & 2 & -1 \end{bmatrix}$$

(b) Transform the matrix to a linear programming problem and solve **6**

$$\begin{bmatrix} 3 & -3 & 4 \\ -1 & 1 & -3 \end{bmatrix}$$