This question paper contains 16+2 printed pages]

Your Roll No. ....

5274

B.A. Prog./III **B**

(L)

MATHEMATICS—Paper III

(Selected Topics in Mathematics)

(Admissions of 2004 and onwards)

Time : 3 *Hours* *Maximum Marks* : 75

(*Write your Roll No. on the top immediately on receipt of this question paper.*)

Note :— The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

Attempt six questions in *all*. **Unit I** and **Unit II** are compulsory and contain *four* questions. In **Unit III** choose any of the options and attempt *two* questions from the same. Marks are indicated against each question

Unit I

1. (*a*) Define neighbourhood of a point. 7

If $S = [0, 1]$, show that S is neighbourhood of every x, $0 < x < 1$, but S is not a neighbourhood of $x = 0$ and $x = 1$.

(*b*) Give *two* examples of each of the following : 6

(*i*) A bounded set

(*ii*) A set which is bounded above but not bounded below.

(*iii*) A set which is bounded below but not bounded above

(*iv*) A set which is neither bounded above nor bounded below.

Or

(*a*) Define limit point of a set. Prove that : 7

(*i*) For set Q of rational numbers, every real number is a limit point.

(*ii*) The set N of natural numbers has no limit point.

(*b*) Which of the following functions are uniformly continuous on their domains : 6

(*i*) $f(x) = x \quad 0 \le x < \infty$

(*ii*) $f(x) = x^2 \quad 0 \le x \le 5$

(*iii*) $f(x) = \frac{1}{x} \quad 0 < x \le 1$

2. (*a*) State Cauchy criterion for convergence of a sequence. Apply it to prove that the sequence $<a_n>$ is not convergent, where : 6

$$a_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

(*b*) Find the sequence of partial sums of the series : 6

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots\dots\dots.$$

Is the series convergent ? If so, find its sum.

Or

(*a*) Show that the sequence $<a_n>$ defined by :

$$a_1 = 1,\ a_{n+1} = \frac{4 + 3a_n}{3 + 2a_n},\ n \ge 1;$$

is bounded and monotonic. Find $\lim_{n \to \infty} a_n$. 6

(*b*) State and prove Cauchy's *n*th root test for convergence of a positive term series. 6

3. (*a*) Show that the improper integral : 6

$$\int_0^\infty \frac{\sin x}{x^p}\, dx$$

is convergent for $1 < p < 2$.

(*b*) Prove that if $f(x)$; defined on $[a, b]$ is integrable on $[a, b]$, then $\forall \in > 0, \exists \delta > 0$ such that :

$$|U(P, f) - L(P, f)| < \in$$

$\forall$ partitions P with $\| P \| < \delta$. 7

Or

(*a*) Define Beta function. Prove that :

$$\int_0^{\pi/2} \sin^p x \cos^q x \, dx = AB(m, n)$$

where :

$$A = 1/2,\ m = \frac{1}{2}(p+1),\ n = \frac{1}{2}(q+1)$$ 7

(*b*) State Weierstrass M-test for uniform convergence of a series of functions. Hence prove that the series : 6

$$\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$$

is uniformly convergent on **R**.

Unit II

(Computer Programming)

4. (*a*) Write a program to evaluate the function :

$$f(x) = 1 + x^2 / 2! + x^4 / 4! - (.50) s G^2 x + (4 - x^2)^{1/2}$$

for a set of values of x. 6

(*b*) Check whether the following are valid or invalid conditional statements. Explain : 7

(*i*) If $(a > b)$ if $(c > d)$ $x = y$; else $x = z$; else $x = w$;

(*ii*) If $(x = y)$ $i = 1$; else if $(x \, ! = y)$ $i = 2$;

Or

(*a*) Write a program to obtain the transpose of an $m \times n$ matrix. 6

(*b*) Write C expressions of the following : 7

(*i*) $$\frac{ab}{c + \frac{dk}{m} + k} + a$$

(*ii*) $a^3 + b^3 + c^3 - 3ab\ (a + b)$

Unit III (1)

(Numerical Analysis)

Note : Use of scientific calculator is allowed.

5. (*a*) Solve :

$$x^4 - x - 10 = 0$$

by Regula-Falsi method correct to three decimal places. 6

(*b*) Solve the following system of equations : 6

$$\begin{aligned} 4x + y + 2z &= 4 \\ 3x + 5y + z &= 7 \\ x + y + 3z &= 3 \end{aligned}$$

by Jacobi and Gauss-Siedel three times iterative schemes starting with the initial vector $X^{(0)} = 0$.

Or

(*a*) Obtain the polynomial approximation to : 6

$$f(x) = (1 - x^2)^{\frac{1}{2}}$$

over [0, 1] by Taylor expansion about $x = 0$.

(*b*) Find the largest eigenvalue of the matrix : 6

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$$

using power method.

6. (*a*) Let :

$$f(x) = \log (1 + x),\ x_0 = 1 \text{ and } x_1 = 1.1$$

Use linear interpolation to calculate an approximate value of $f(1.04)$. 6

(*b*) Compute the double integral : 6

$$\int_0^1 \left(\int_1^2 \frac{2^{xy}}{(1 + x^2)\ (1 + y^2)}\, dy \right) dx$$

Or

(*a*) Obtain the approximate value :

$$I = \int_{-1}^{1} e^{-x^2} \cos x \quad dx$$

using Radau integration method for $n = 2, 3$. 6

(*b*) Use the method of least square to fit the curve :

$$y = \frac{C_0}{x} + C_1 \sqrt{x}$$

to the table values : 6

x	y
0.1	21
0.2	11
0.4	7
0.5	6
1	5
2	6

Unit III (2)

(Discrete Mathematics)

5. (*a*) Define the following :

   (*i*) A path in a graph

   (*ii*) A connected graph

   (*iii*) An Eulerian path

   (*iv*) A directed graph. 6

   (*b*) If G is a connected planar graph with e edges, v vertices and f faces. then prove that : 6

$$f = e - v + 2$$

Or

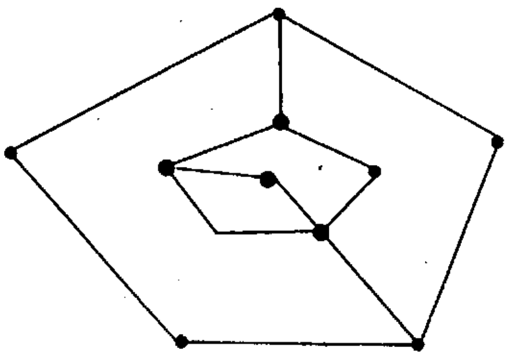
   (*a*) Define the following giving examples : 6

   (*i*) The complement of a sibgraph of a graph G

   (*ii*) A Hamiltonian path

   (*iii*) A directed multigraph.

(*b*) Show that the graph in the following figure has no Hamiltonian circuit : 6



6. (*a*) Let :

$$E(x_1, x_2, x_3, x_4) = (x_1 \wedge x_2 \wedge \bar{x}_3) \vee (x_1 \wedge \bar{x}_2 \wedge x_4) \wedge (x_2 \wedge \bar{x}_3 \wedge \bar{x}_4)$$

be a Boolean expression over the two valued Boolean algebra. Write $E(x_1, x_2, x_3, x_4)$ in both disjunctive and conjunctive nomal forms. 6

(*b*) Show that the following statement is a tautology : 6

$$(A \rightarrow B) \rightarrow [A \rightarrow (A \wedge B)]$$

Or

(*a*) Write Boolean functions corresponding to the Boolean expressions : 6

(*i*) $\left(\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3\right) \vee \left(\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3\right)$

$\vee \left(x_1 \wedge x_2 \wedge x_3\right)$

(*ii*) $\left(x_1 \wedge \bar{x}_2 \wedge \bar{x}_3\right) \vee \left(\bar{x}_1 \wedge x_2 \wedge \bar{x}_3\right)$

$\vee \left(x_1 \wedge x_2 \wedge \bar{x}_3\right)$

in tabular form specifying values of f corresponding to various possible combinations of values x_1, x_2 and x_3.

(*b*) Express the function in the following table in disjunctive normal form and conjunctive normal form : 6

	f
(0, 0, 0)	1
(0, 0, 1)	0
(0, 1, 0)	1
(0, 1, 1)	0
(1, 0, 0)	0
(1, 0, 1)	1
(1, 1, 0)	0
(1, 1, 1)	1

Unit III (3)

(Mathematical Statistics)

5. (*a*) Show that for any discrete distribution, standard deviation is not less than mean deviation from mean. 6

(*b*) If X and Y are random variables with correlation coefficient p between them, then show that : 6

$$u = x \sin \alpha + y \cos\alpha$$
$$v = y \sin \alpha - x \cos \alpha$$

are uncorrelated if :

$$\tan 2\alpha = \frac{2p\sigma \times \sigma_y}{\sigma_y^2 - \sigma_x^2}$$

Or

(*a*) For a binomial distribution, find β_1 and γ_1. 6

(*b*) Show that Poisson distribution is a limiting case of the binomial distribution. 6

6. (*a*) For a discrete distribution, prove that the mean deviation about mean $\bar{x}$ can be written in the form : 6

$$\frac{2}{N}\left[\bar{x}\sum_{x_L < \bar{x}} f_i - \sum_{x_L < \bar{x}} f_i\, x_i\right]$$

where f_i is the frequency of the value x_i and N is the total frequency.

(*b*) Fit a second degree parabola to the following data : 6

x	y
0	1
1	5
2	0
3	22
4	38

Or

(*a*) Show that the arithmetic mean of the regression coefficient is greater than the correlation coefficient. 6

(*b*) Let X be a random variable and c, a constant. Show that :

$$E(X - c)^2 = \text{var}(x) + (E(x) - c)^2 \qquad 6$$

Unit III (4)

(Mechanics)

5. (*a*) Like parallel forces P, Q, R act at the vertices of ΔABC. If their resultant passes through the orthocentre of the triangle for all directions of the forces, show that : 6

$$P : Q : R = \tan A : \tan B : \tan C.$$

(*b*) Find the centre of gravity of a loop of the lamniscate : 6

$$r^2 = a^2 \cos 2\theta$$

Or

(*a*) Find the centre of pressure of a triangular area when one angular point A is in free surface. 6

(*b*) Three forces P, Q, R act along the sides of the triangle formed by the lines : 6

$$x + y = 1, y - x = 1, \ y = 2$$

Find the equation of the line of action of their resultant.

6. (*a*) If particles are projected from the point 'O' in a vertical plane under gravity with velocity $\sqrt{29k}$, prove that locus of the vertices of their paths is the ellipse : 6

$$x^2 + 4y(y - k) = 0$$

(*b*) An automobile travels round a curve of radius '*r*'. If '*h*' is the height of the centre of gravity above the ground and 2*b* the width between the wheels, show that it will overturn if the speed exceeds : 6

$$\sqrt{\frac{9rb}{h}}$$

Assuming no side slipping takes place.

Or

(*a*) Obtain the energy of equation of the central orbit. 6

(*b*) The displacement of a moving point at any point is given by :

$$x = a\cos kt + b\sin kt$$

Show that the point executes a simple harmonic motion. 6

Unit III (5)

(Theory of Games)

5. (*a*) Using simplex method solve the following linear programming problem : 6

Maximize : $z = 5x_1 + 10x_2 + 8x_3$

Subject to $3x_1 + 5x_2 + 2x_3 \leq 60$

$4x_1 + 4x_2 + 4x_3 \leq 72$

$2x_1 + 4x_2 + 5x_3 \leq 100$

$x_1, x_2, x_3 \geq 0$

(*b*) Give the dual of the following linear programming problem : 6

Minimize : $z = 2x_1 + 3x_2 + 4x_3$

Subject to $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 = 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2 \geq 0$ and x_3 is unrestricted.

Or

(*a*) Solve the following linear programming problem by graphical method : 6

Minimize : $z = 2x_1 + x_2$

Subject to $5x_1 + 10x_2 \leq 50$

$x_1 + x_2 \geq 1$

$x_1 \leq 4$

$x_1, x_2 \geq 0$

(*b*) Use Big M method to solve the following linear programming problem : 6

Minimize : $z = 12x_1 + 20x_2$

Subject to $6x_1 + 8x_2 \geq 100$

$7x_1 + 12x_2 \geq 120$

$x_1, x_2 \geq 0$

6. (*a*) Let f be a real valued function such that $f(x, y)$ is defined wherever $x \in A$ and $y \in B$ and suppose that :

$$\max_{x \in A} \min_{y \in B} f(x, y)$$

and

$$\min_{y \in B} \max_{x \in A} f(x, y)$$

both exist. Let (x_0, y_0) be a Saddle point of the function. Prove that : 6

$$f(x_0, y_0) = \max_{x \in A} \min_{y \in B} f(x, y) = \min_{y \in B} \max_{x \in A} f(x, y)$$

(*b*) Find the range of values for p and q that will render the entry (2, 2) a saddle point in the following game : 6

$$A \begin{bmatrix} 1 & q & 6 \\ p & 5 & 10 \\ 6 & 2 & 3 \end{bmatrix}$$

Or

(*a*) Use linear programming to solve the following game : 6

$$\begin{array}{c} \quad A \\ B \begin{bmatrix} 2 & -2 & 3 \\ -3 & 5 & -1 \end{bmatrix} \end{array}$$

(*b*) Solve the following game problem graphically : 6

$$\begin{array}{c} \quad B \\ A \begin{bmatrix} 4 & -2 & 3 & -1 \\ -1 & 2 & 0 & 1 \end{bmatrix} \end{array}$$