This question paper contains 12 printed pages.]

Your Roll No. ....

5273

B.A. (Programme) / III B

(T)

MATHEMATICS – Paper III

(Selected Topics in Mathematics)

(Admissions of 2004 and onwards)

Time : 3 Hours **Maximum Marks : 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note : The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

Instructions : Attempt **all** questions selecting **two** parts from each question. **Unit-I** and **Unit-II** are compulsory and contain **four** questions. In **Unit-III**, choose any of the options and attempt **two** questions from the same. Marks are indicated.

UNIT – I

1. (a) State the properties which prove that the set R of real numbers is a complete ordered field. 6

(b) Define any two of the following illustrating each by means of an example :

(i) Supremum of a set.

(ii) Neighbourhood (nhd) of a point.

(iii) Limit point of a set. 6

(c) Prove that if a function f is continuous on [a, b] and f(a) f(b) < 0, then $\exists$ some point $C \in]a, b[$ such that f(c) = 0. 6

(d) Discuss the type of discontinuity if any of the function defined on [0, 1] as follows : 6

$$f(0) = 0,\ f(x) = \frac{1}{2} - x,\ \text{if } 0 < x < \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{2},\ f(x) = \frac{3}{2} - x,\ \text{if } \frac{1}{2} < x < 1$$

$$f(1) = 1.$$

2. (a) Prove that every function uniformly continuous on an interval is also continuous on that interval. What about the converse ? Justify by an example. 7

(b) State Cauchy's General Principle of Convergence for real sequences and prove that the sequence $<a_n>$ given by

$a_n = 1 + \frac{1}{3} + \frac{1}{5} + \ldots\ldots + \frac{1}{2n-1}$ does not converge. 7

(c) State and prove D'Alembert's Ratio Test for positive term series. 7

(d) Test for convergence any two of the following series : 7

(i) $\frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \ldots \; (x > 0)$

(ii) $\sum_{n=1}^{\infty} \frac{1.2.3. \ldots n}{7.10. \ldots (3n+4)}$

(iii) $\sum_{n=1}^{\infty} \frac{1.3.5. \ldots (2n-1)}{2.4.6. \ldots 2n} \frac{x^{2n+1}}{2n+1} \; (x > 0)$

3. (a) Test for uniform convergence of the sequence of functions $\{fn\}_{n=1}^{\infty}$ where

$$fn(x) = \frac{nx}{1 + n^2 x^2}, x \in [0, 1].$$

State clearly any result you are using. 6½

(b) Show that the function defined on [0, 1] defined by,

$$f(x) = \frac{1}{2n}, \frac{1}{2^{n+1}} < x \leq \frac{1}{2^n} \quad n = 0, 1, \ldots\ldots$$

$$= 0 \quad \text{if } x = 0$$

is Riemann integrable. 6½

(c) Test the convergence of $\int_0^{\infty} e^{-x} x^{n-1} \, dx$. 6½

Unit – II

(Computer Programming)

4. (a) (i) What do you mean by floating point constant ? What are the rules for writing floating point constants ? 3

(ii) Given an integer, write a program to reverse and print it. For example, if the given number is 12386, the number printed should be 68321. 3

(b) (i) Give the general form of the for loop. Explain how it works with an example. 3

(ii) Write functions to add and subtract two complex numbers (a + ib) and (c + id). 3

(c) (i) Write a short note on the continue statement. Explain with example, how it is used ? 3

(ii) Write a program to transpose the following matrix :

$$A = \begin{pmatrix} -9 & 6 & 4 \\ 4 & 3 & 5 \\ 9 & 2 & 1 \end{pmatrix}$$ 3

Unit – III (1)

(Numerical Analysis)

(Use of scientific calculator is allowed)

5. (a) Find the iterative method based on the Newton – Raphson method for finding $\sqrt{N}$, where N is a positive real number. Apply the method to N = 18, to obtain the result correct to two decimals. 6

(b) Using Gaussian Elimination method, solve the system of equations :

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

6

(c) If A is a strictly diagonally dominant matrix, then show that the Jacobi iteration scheme converges for any initial starting vector. 6

6. (a) The following values of the function $f(x) = \sin x + \cos x$ are given :

x :	10°	20°	30°
$f(x)$:	1.1585	1.2817	1.3660

Construct the quadratic Lagrange interpolating polynomial that fits the data. Hence find $f(\pi/12)$.

Use formula $R = \pi\theta/180$, where θ is the angle in degrees and R is the corresponding angle in radians. 6

(b) Evaluate the integral $\int_{-1}^{1} x^2 \ e^{-x} \ dx$ by Simpson's $\frac{1}{3}$ rule with spacing $h = 0.25$. 6

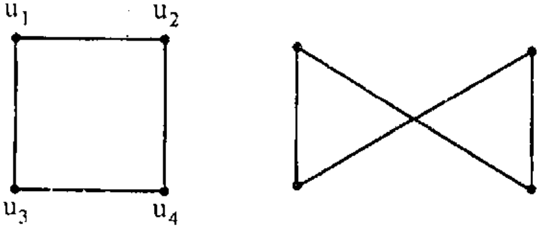
(c) Evaluate the integral of $\frac{\sin x}{x}$ between $x = 0$ and $x = 1$, with a four-term Gaussian formula. 6

Unit – III (2)

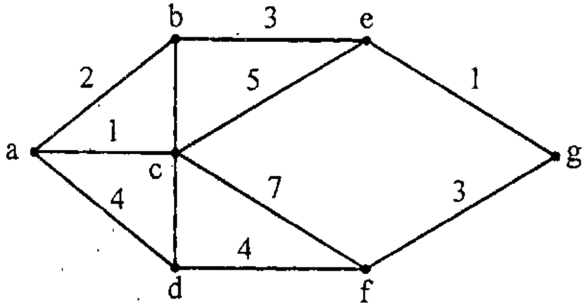
(Discrete Mathematics)

5. (a) Show that there is 0 simple path between every pair of distinct vertices of a connected undirected graph. 6

(b) Show that the following graphs are isomorphic. 6



(c) Determine a shortest path between a and z in the graph using Dijhstra algorithm. 6

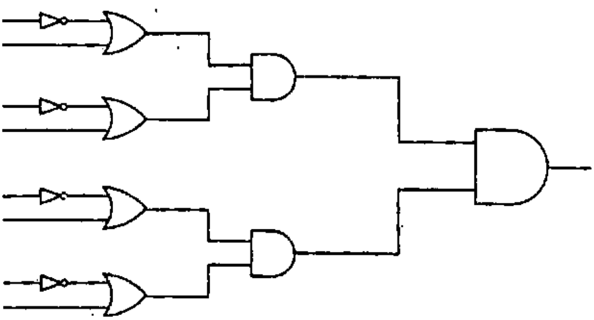


6. (a) Define the meaning of conjunctive normal form of a Boolean expression. Express the following Boolean expression.

$E\ (x_1,\ x_2,\ x_3) = (x_1 \wedge x_2) \vee (x_1 \wedge x_3) \vee (x_2 \wedge x_3)$

over the two valued Boolean algebra into conjunctive normal form. 6

(b) Write the Boolean expression corresponding to the circuit. 6



(c) Show how an AND-gate can be replaced by a suiable interconnection of OR-gates and NOT-gates. 6

Unit – III (3)
(Mathematical Statistics)

5. (a) For a group of 200 candidates, the mean and standard deviation of scores were found to be 40 and 15 respectively. Later on it was discouraged that the scores 43 and 35 were misread as 34 and 53 respectively. Find the corrected mean and standard deviation corresponding to the corrected figures. 6

(b) Let u = aX + bY and v = bX – aY, where X and Y are measured from their means. If the correlation coefficient between X and Y in P and u and v are uncorrelated, prove that

$$\sigma_u^2 + \sigma_v^2 = (a^2 + b^2)(\sigma_X^2 + \sigma_Y^2).$$ 6

(c) Fit a parabola to the following data taking X as independent variable : 6

X :	1	2	3	4	5
Y :	1.8	5.1	9.0	14	19

6. (a) There are five boxes, they are numbered 1 to 5. Each box contains 10 balls. Box i has i defective balls and 10-i non-defective balls. Suppose a box is selected at random and then a ball is selected at random from the selected box. What is the probability that the ball selected is defective ? If the selected ball is defective, what is the probability that it came from box 5 ? 3 + 3 = 6

(b) A coin is tossed until a head appears. What is the expectation of the number of tosses required ? 6

(c) If the skulls are classified A, B, C according as the length, breadth index is under 75, between 75 and 80 and over 80, find approximately (assuming that the distribution is normal) the mean and standard deviation of a series in which A are 58% B are 38% and c are 4% being given that if 6

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{\frac{-x^2}{2}} dx.$$

then, f(0.20) = 0.08 and f(1.75) = 0.46.

Unit – III (4)

(Mechanics)

5. (a) A door of weight w, height 2a, and width 2b, is hinged at the top and bottom. If the reaction at the upper hinge has no vertical component, find the components of reaction on both hinges. Assume that the weight of the door acts at its centre. 6

(b) Find the mass centre of a wire bent into the form of an isosceles right-angled triangle. 6

(c) A body consisting of a cone and a hemisphere on the same base rests on a horizontal table, the hemisphere being in contact with the table. Show that the greatest height of the cone so that the equilibrium is stable is $\sqrt{3}$ times the radius of the sphere. 6

6. (a) Prove that if tangential and normal components of acceleration of a particle describing a plane curve be constant throughout the motion, the angle Ψ through which the direction of motion turns in time 't' is given by 6

$\Psi = A \log (1 + Bt)$, where A, B are constants.

(b) If the bed of a simple pendulum is projected from the position of stable equilibrium with velocity equal to that due to falling from the highest point of the circle, show that the time of describing any angle 'θ' is

$$\sqrt{\frac{l}{g}} \log\left(\sec\frac{\theta}{2} + \tan\frac{\theta}{2}\right)$$ 6

(c) Establish the formula

$$\frac{d^2u}{d\theta^2} + u = \frac{F}{h^2u^2}, \theta = hu^2, \text{ where } u = \frac{1}{r}$$

for the motion of a particle describing a central orbit under an attraction F per unit mass. 6

Unit – III (5) Theory of Games

5. (a) What are the characteristics of the standard form of a linear programming problem. Reduce the following linear programming problem to the standard form : 6

Determine $x_1, x_2, x_3 \geq 0$ so as to maximize $A = 2x_1 + x_2 + 4x_3$, subject to the constraints

$$2x_1 + 4x_2 \leq 4$$

$$x_1 + 2x_2 + x_3 \geq 5$$

$$2x_1 + 3x_3 \leq 2$$

(b) Solve the following LPP using two phase or Charne's penalty method : 6

Minimize $Z = x_1 - 2x_2 - 3x_3$

Subject to $-2x_1 + x_2 + 3x_3 = 2$

$$2x_1 + 3x_2 + 4x_3 = 1$$

$$x_1, x_2, x_3 \geq 0$$

(c) Write down the dual of the LPP : 6

Maximize $Z = x_1 + x_2 + x_3$

Subject to $x_1 - 3x_2 + 4x_3 = 5$

$$x_1 - 2x_2 \leq 3$$

$$2x_2 - x_3 \geq 4$$

$x_1, x_2 \geq 0$, x_3 unrestricted

6\. (a) Solve graphically the rectangular game whose payoff matrix is : **6**

$$\text{Player A} \begin{array}{c} \\ A_1 \\ A_2 \end{array} \overset{\text{Player B}}{\begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \\ \end{array}} \begin{bmatrix} 2 & 2 & 3 & -2 \\ 4 & 3 & 2 & 6 \end{bmatrix}$$

(b) State the general rule of dominance for two person zero sum game. Use dominance principal to solve the following game whose payoff matrix is **6**

$$\text{Player A} \overset{\text{Player B}}{\begin{bmatrix} 1 & -1 & 0 \\ -6 & 3 & -2 \\ 8 & -5 & 2 \end{bmatrix}}$$

(c) Transform the following matrix game into their corresponding primal and dual L.P.P.

$$\begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \end{bmatrix}$$

Hence solve them. **6**