[This question paper contains 12 printed pages.]

1184 Your Roll No. ....

B.A. (Programme) / III C

(T)

MATHEMATICS — Paper III

(Selected Topics in Mathematics)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :— The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat 'A'). These marks will however be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

Attempt **all** *questions selecting* **two** *parts from each question.* **Unit-I** *and* **Unit-II** *are compulsory and contain* **four** *questions. In* **Unit-III**, *choose any of the options and attempt* **two** *questions from the same. Marks are indicated.*

P.T.O.

UNIT – I

1. (a) State the properties which prove that the set $\mathbb{R}$ of real numbers is a complete ordered field. (6)

(b) Define any three of the following also give example of each :–

(i) Supremum of a set

(ii) Infimum of a set

(iii) Limit point of a set

(iv) Neighbourhood of a point (6)

(c) Show that the function f defined on $\mathbb{R}$ by

$$f(x) = \begin{cases} 1 & \text{when } x \text{ is rational} \\ -1 & \text{when } x \text{ is irrational} \end{cases}$$

is discontinuous at every real number. (6)

(d) Show that the function $f(x) = \frac{1}{x^2}$ is uniformly continuous on $[a, \infty)$ where $a > 0$. Also show that f is not uniformly continuous on $(0, \infty)$. (6)

2. (a) Define a Cauchy sequence and prove that every Cauchy sequence converges. Hence show that the sequence (a_n) where

$$a_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$$

is not a Cauchy sequence. (7)

(b) State and prove Cauchy's n^{th} root test for the convergence of positive term series. (7)

(c) Test for convergence for any **two** of the following series :-

(i) $\sum \left(1 - \frac{1}{n}\right)^{n}$

(ii) $\sum \frac{1 \,.\, 3 \,.\, 5 \ldots (2n-1)}{2 \,.\, 4 \,.\, 6 \ldots 2n} x^{n} \quad \forall\, x > 0$

(iii) $\sum \sin \frac{1}{n^{2}}$ (7)

(d) State Leibnitz Test for the convergence of alternating series and hence check the convergence of the series

$$\sum \frac{(-1)^{n-1}}{n}$$ (7)

3. (a) Show that the function f(x) defined on the interval [0, 3] as

$$f(x) = [x^{2}] \quad \forall\, x \in [0, 3]$$

is Riemann Integrable. State clearly any result you are using. (6)

(b) Discuss the convergence of the following improper integrals :

(i) $\int_0^{\infty} \frac{x^{m-1}}{1+x}\,dx$

(ii) $\int_0^{1} \frac{dx}{\sqrt{x - x^2}}$ (6)

(c) Find the Fourier series of the function

$$f(x) = \begin{cases} x+\pi & \text{for } -\pi < x < 0 \\ \pi - x & \text{for } 0 < x < \pi \end{cases}$$ (6)

(d) Show that the sequence (f_n) where

$$f_n(x) = \frac{nx}{1+n^2x^2},$$

is not uniformly convergent on any interval containing zero. (6)

UNIT – II

(Computer Programming)

4. (a) (i) Write C expressions of the following

(1) $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ (2) $e = mc^2$ (3)

(ii) Explain logical operators with suitable example for each of them. (3½)

(b) (i) Pick the illegal conditional statement from the following. Explain why they are illegal

(1) if a == b x = y; else x = z;

(2) if (x == y) i = 1; else if (x != y) i = 2;

(3) if (z == a + b + c) {x = x + 1; y = y + 2;}

(3)

(ii) Write a short notes on switch statement. Explain its use with an example. (3½)

(c) (i) Write a program to determine wheather a number is 'odd' or 'even' and print the message

NUMBER is ODD

OR

NUMBER is EVEN (3½)

(ii) Explain the need for array variables. (3)

UNIT – III (I)

(Numerical Analysis)

(Use of scientific calculator is allowed)

5. (a) Describe bisection method to find a root of an equation $f(x) = 0$. (6)

(b) Use Gauss elimination to solve

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

(6)

P.T.O.

(c) For the following system of equations :

$$\begin{pmatrix} 5 & 1 & 2 \\ 3 & 4 & -1 \\ 2 & 3 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 10 \end{pmatrix}$$

Starting with $x_1 = x_2 = x_3 = 0$, using Gauss Seidel method, find the solution after performing three iterations. (6)

6. (a) Evaluate

$$\int [illegible] e^{[illegible]} dx$$

using Simpson's $\frac{3}{8}$ rule with 6 sub-intervals. (6)

(b) Calculate the nth divided difference of $\frac{1}{x}$, based on the points $x_0, x_1, x_2, ----, x_n$. (6)

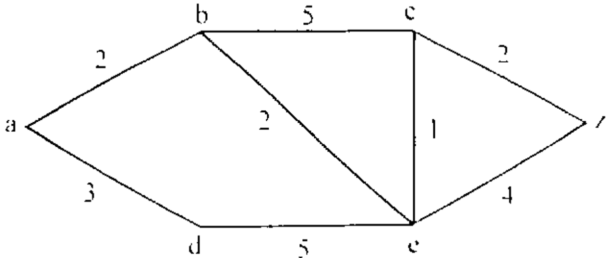
(c) Given $p(0) = 1$, $p(1) = 3$, $p(3) = 55$. Find the Lagrange quadratic interpolating polynomial, which fits the given data. Estimate $p(1.5)$. (6)

UNIT – III (2)

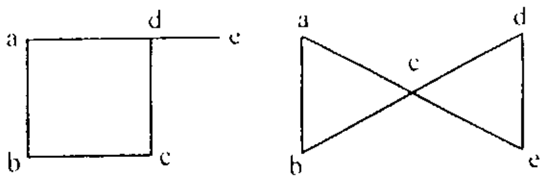
(Discrete Mathematics)

7. (a) Define Königsberg bridge problem. State the theorem that solves the problem. (6)

(b) Find the length of a shortest path between 'a' and 'z' in the weighted graph. (6)



(c) Show the neither of the graphs shown below has a Hamiltonian circuit. (6)



6. (a) Express the function f given below in conjuctive normal form :

			f
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

(6)

(b) Draw the switching circuit corresponding to the algebraic expression :

a PAR (b SER d) PAR (c' SER d) (6)

(c) Show how an 'OR' gate can be replaced by a suitable interconnection of AND gates and NOT gates. (6)

UNIT – III (3)

(Statistics)

5. (a) The first four moments of a distribution about the value 4 are −1.5, 17, −30, 108. Find the moments about the mean β_1 and β_2. (6)

(b) Show that for the Poisson distribution with mean λ,

$$\mu_{r+1} = r\lambda\mu_{r-1} + \lambda \frac{d\mu_r}{d\lambda}.$$

Hence deduce the values of β_1 and β_2. (6)

OR

Deduce the first four moments about the mean of the Poisson distribution from those of binomial distribution. (6)

(c) Fit a straight line to the following data :

x :	0	5	10	15	20	25
y :	12	15	17	22	24	30

(6)

6. (a) Three groups of students contain respectively 3 female and one male, 2 female and 2 male, and 1 female and three male. One student is selected at random from each group. Show that the chance that the three students selected consists of 1 female and two male is 13/32. (6)

(b) If X is a normal variate with mean 30 and standard deviation 5. Find the probabilities that :

(i) $26 \le X \le 40$, (ii) $X \ge 45$, and (iii) $|X - 30| > 5$ (6)

(c) If x_1 and x_2 are two independent Poisson variables with parameters m_1 and m_2, then show that $x_1 + x_2$ is a Poisson variable with parameter $m_1 + m_2$. What can you say about $x_1 - x_2$? (6)

UNIT – III (4)

(Mechanics)

5. (a) If forces of magnitude P, Q, R act at a point parallel to and in the directions of the sides BC, CA and AB respectively of a triangle ABC, prove that the magnitude of the resultant is

$$[P^2 + Q^2 + R^2 - 2QR\cos A - 2RP\cos B - 2PQ\cos C]^{1/2}$$

(6)

(b) Two equal beams AB and AC, connected by a hinge at A, are placed in a vertical plane with their extremities B and C, resting on a horizontal plane; they are kept from falling by strings connecting B and C with the middle points of the opposite beams. Show that the ratio of the tension of each string to the weight of a beam is

$$\frac{1}{8}\sqrt{9\cot^2\theta + 1}$$

θ being the inclination of each beam to the horizon. (6)

(c) A uniform beam of length 2a, rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance b from the wall. Show that in the position of equilibrium, the beam is inclined to the wall at an angle $\sin^{-1}\left(\frac{b}{a}\right)^{1/3}$. (6)

6\. (a) A simple pendulum of mass m and length a is hanging in equilibrium. At time t = 0 a small horizontal disturbing force X comes into operation and continues to act, varying with time according to the formula

$$X = m\,b\sin 2pt$$

where $p^2 = g/a$. Find a formula giving the position of the pendulum at any time. (6)

(b) A particle is placed on the outside of a smooth vertical circle. If the particle starts from a point whose angular distance is α from the highest point of circle, show that it will fly off the curve when

$$\cos\theta = \frac{2}{3}\cos\alpha$$ (6)

(c) Find the law of force, if a particle moving under a central attractive force to a point, describes the conic

$$\frac{l}{r} = 1 + e\cos\theta.$$ (6)

UNIT – III (5)

(Game Theory)

5. (a) Solve the following Linear Programming Problem Graphically :

Maximize $Z = 4x + 6y$

Subject to the constraints :

$$x + y = 5$$
$$x \geq 2$$
$$y \geq 4, \quad x, y \geq 0$$ (6)

(b) Solve the following Linear Programming Problem by Simplex method :

Maximize $Z = 2x_1 + x_2 - x_3$

Subjec to the constraints :

$$x_1 + x_2 \leq 1$$
$$x_1 - 2x_2 - x_3 \leq -2$$
$$x_1, x_2, x_3 \leq 0$$ (6)

(c) Write the dual of the following Linear Programming Problem :

Maximize $Z = 20x_1 + 15x_2 + 18x_3 + 10x_4$

Subject to the constraints :

$$4x_1 + 3x_2 + 10x_3 + x_4 \le 60$$
$$x_1 + x_2 + x_3 = 27$$
$$x_1 + 4x_3 + 7x_4 \ge 35$$
$$x_1, x_2, x_3 \ge 0, \; x_4 : \text{unrestricted}$$

in sign (6)

6. (a) Solve the following game by Dominance method

Player A
$$\text{Player B}\begin{bmatrix} 1 & 3 & 2 & 7 & 4 \\ 3 & 4 & 1 & 5 & 6 \\ 6 & 5 & 7 & 6 & 5 \\ 2 & 0 & 6 & 3 & 1 \end{bmatrix}$$
(6)

(b) Solve the following game Graphically :

Player A
$$\text{Player B}\begin{bmatrix} 1 & 3 & 3 & 7 \\ 2 & 5 & 4 & 6 \end{bmatrix}$$
(6)

(c) Solve the following Game by using simplex method :

Player A
$$\text{Player B}\begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & 3 \\ 6 & 2 & -2 \end{bmatrix}$$
(6)

(200)