[This question paper contains 12 printed pages.]

1185 *Your Roll No*

B.A. Prog. / III **C**

(L)

MATHEMATICS – Paper III

(Selected Topics in Mathematics)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :– The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

Attempt **all** *questions selecting* **two** *parts from each question.* **Unit I** *and* **Unit II** *are compulsory and contain* **four** *questions. In* **Unit III** *choose any of the option and attempt* **two** *questions from the same. Marks are indicated.*

UNIT – I

1. (a) For any two real numbers x and y; show that

$$\frac{|x+y|}{1+|x+y|} \le \frac{|x|}{1+|x|} + \frac{|y|}{1+|y|} \qquad (6)$$

(b) Let S be a non empty set of real numbers which is bounded above. Show that a real number $u = \text{Sup } S$ if and only if

(i) $x \le u \quad \forall\ x \in S$

(ii) For each $\in > 0$ there exists $x_0 \in S$ such that $x_0 > u - \in$. (6)

(c) Show that the function f defined on $\mathbb{R}$ by

$$f(x) = [x]$$

is continuous everywhere except at integers. (6)

(d) Let f be a real valued function defined on an interval I. Prove that if f is continuous at C then $|f|$ is also continuous at C. Give an example to justify that converse need not be true. (6)

2. (a) State and prove D'Alembert's Ratio Test for the convergence of the term series. (7)

(b) Let (a_n) be a sequence defined by

$$a_1 = \sqrt{7}, \quad a_{n+1} = \sqrt{7 + a_n} \quad n > 1$$

show that (a_n) is convergent and find its limit. (7)

(c) State Cauchy's Principle of convergence. Show that the series

$$1+\frac{1}{4}+\frac{1}{7}+----+\frac{1}{3n-2}+---$$

is not convergent. (7)

(d) Test for convergence for any **two** of the following series :–

(i) $\sum u_n$ where $u_n = \sin\frac{\pi}{2^n}$

(ii) $\sum u_n$ where $u_n = \frac{2^{n-1}}{3^n+1}$

(iii) $\sum u_n$ where $u_n = \frac{1.3.5---(2n-1)\,x^n}{2.4.6---2n}$, $x>0$

(7)

3. (a) Show that every continuous function defined on [a, b] is Riemann Integrable. What about the converse ? Justify your answer. (6)

(b) Let f be defined in [a, b] as follows :

$$f(x) = \begin{cases} 0 & \text{when x is rational} \\ 1 & \text{when x is irrational} \end{cases}$$

Show that f is bounded in [a, b] but is not Riemann Integrable therein. (6)

(c) Discuss the convergence of following improper integrals

(i) $\int_0^{\pi} \frac{\sqrt{x}}{\sin x}\, dx$

(ii) $\int_0^{\infty} \frac{\sin^2 x}{x^2}\, dx$ (6)

(d) Find the radius of convergence of the series :

(i) $x + \frac{x^2}{2^2} + \frac{2!}{3^3} x^3 + \frac{3!}{4^4} x^4 + - - -$

(ii) $\sum \frac{2^n x^n}{n!}$ (6)

UNIT – II

(Computer Programming)

4. (a) (i) What is the general form of switch statement ? Explain with an example. (3)

(ii) Write a program to find the sum of squares of elements on the diagonal of a square matrix. (3½)

(b) (i) Pick the incorrect identifiers from the following. Explain why they are incorrect.

(1) double (2) Rs – ps (3) int result (3½)

(ii) Evaluate the following expressions :

float a = 2.5, b = 2.5;

(1) a + 2.5 / b + 4.5 (2) (a + 2.5) / b − 4.5

(3)

(c) (i) The following is a segment of a program

```
x = 1;
y = 1;
if (n > 0)
x = x + 1
y = y − 1
printf("%d %d", x, y)
```

What will be values of x and y if n assumes a value of (a) 1 (b) 0. (3,2)

(ii) Write a program that will read the value of x and evaluate the following function

$$y = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -1 & \text{for } x < 0 \end{cases}$$ (3)

UNIT – III (1)

(Numerical Analysis)

(Use of scientific calculator is allowed.)

5. (a) Using Gaussian Elimination method, solve the system of equations

$$5x - 2y + z = 4$$

$$7x + y - 5z = 8$$

$$3x + 7y + 4z = 10 \quad (6)$$

(b) Perform three iterations of the Newton's method to find the smallest positive root of equation

$$x^4 - x - 10 = 0 \quad (6)$$

(c) Find the root of the equation

$$\sin x - \cos nx + 1 = 0$$

correct to 4 decimal places, using Regula-falsi method. The root lies between 1 and 2. (6)

6. (a) Evaluate $\int_0^1 \frac{dx}{\sqrt{x^3 + 1}}$

using Simpson's one third rule with 10 subintervals. (6)

(b) Use Newton's divided difference formula to find f(x) from the following data :

x :	0	2	3	4	6	7
f(x) :	0	8	0	−72	0	1008

(6)

(c) Evaluate the integral of e^x between x = 0 and x = 1, using three term Gaussian quadrature. (6)

UNIT – III (2)
(Discrete Mathematics)

5. (a) Define the following giving examples

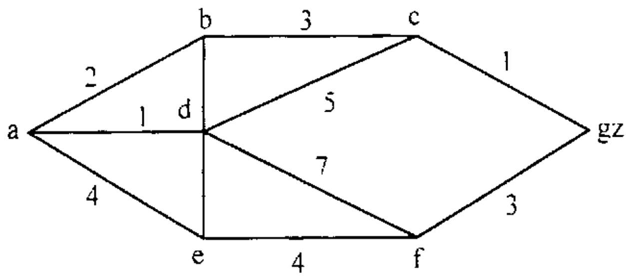
   (i) The complement of a subgraph of a graph G

   (ii) An Eulerian Circuit

   (iii) A connected graph (6)

   (b) Define Konigsberg bridge problem. State the theorem that solves this problem. (6)

   (c) Determine the shortest path between 'a' and 'z' in the following graph (6)



6. (a) Express the boolean expression

   $E(x_1, x_2, x_3) = (x_1 \wedge x_2) \vee (x_1 \wedge x_3) \vee (\overline{x}_2 \wedge x_3)$

   into disjunctive normal form. (6)

   (b) Show how an OR-gate can be replaced by a suitable interconnection of AND-gates and NOT-gates. (6)

(c) Draw the switching circuit corresponding to the algebraic expression :

$$a \text{ PAR } (b \text{ SER } d) \text{ PAR } (\overline{c} \text{ SER } d) \quad (6)$$

UNIT – III (3)

(Statistics)

5. (a) A distribution consists of three components with frequencies of 200, 250 and 300 having means 25, 10 and 15 and standard deviations of 3, 4 and 5 respectively. Find the mean and standard deviation of the combined distribution. (6)

(b) If $z = ax + by$ and r is the correlation coefficient between x and y, show that

$$\sigma_z^2 = a^2\sigma_x^2 + b^2\sigma_y^2 + 2abr\,\sigma_x\sigma_y \quad (6)$$

(c) Fit a second degree parabola to the following data :

x :	0	1	2	3	4
y :	1	5	10	22	38

(6)

6. (a) A can hit a target 3 times in 5 shots, B can hit the same target 2 times in 5 shots and C 3 times in 4 shots. Find the probability of the target being hit when all of them try. (6)

(b) Starting with the identity

$$\sum_{x=0}^{n} {}^{n}c_x \, p^x q^{n-x} = (q + p)^n .$$

Find the mean and variance of Binomial Distribution. (6)

(c) Show that

If $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$

then $X_1 + X_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$,

$X_1 - X_2 \sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$ (6)

UNIT – III (4)

(Mechanics)

5. (a) Two light rings can slide on a rough horizontal rod. The rings are connected by a light inextensible string of length a, to the mid-point of which is attached a weight W. *Show that the greatest* distance between the rings, consistent with the equilibrium of the system, is

$$\frac{\mu a}{\sqrt{1 - \mu^2}}$$

where μ is the coefficient of friction between either ring and the rod. (6)

(b) A square framework, formed of uniform heavy rods of equal weight W jointed together, is hung up by one corner. A weight W is suspended from each of the three lower corners and the shape of the square is preserved by a tight rod along the horizontal diagonal. Find the thrust of the light rod. (6)

(c) Find the centre of mass of the area bounded by the curve $x^{2/3} + y^{2/3} = a^{2/3}$, lying in the positive quadrant. (6)

6. (a) A train moves from rest at 0 to rest at C. From O to A, threre is constant acceleration, the velocity at A being V, from A to B the train moves with the constant velocity V, from B to C there is constant retardation. The distance OA = a, OC = l, BC = b, find the time taken for the journey. (6)

(b) A body moving with S.H.M. has an amplitude a and time period T. Show that the velocity V at a distance x from the mean position is given by

$$V^2T^2 = 4\pi^2(a^2 - x^2). \quad (6)$$

(c) A body of mass M is projected vertically upward in a medium for which the resistance is mK^2V^2. If the initial velocity is V_0, show that the body returns to the point of projection with a velocity V_1, such that

$$V = \frac{V_0}{\left(1 + \frac{K^2 V_0^2}{g}\right)^{1/2}}. \qquad (6)$$

UNIT – III (5)
(Game Theory)

5. (a) State the general Linear Programming Problem in standard form. Rewrite in standard form the following Linear Programming Problem :

Maximize $z = 3x + 2y$

Subject to the constraints :

$$4x + 3y \leq 60$$
$$x \geq 3$$
$$y \geq 2x, \quad x, y \geq 0 \qquad (6)$$

(b) Solve the following Linear Programming Problem by Simplex method.

Maximize $z = 5x_1 + 3x_2$

Subject to the constraints :

$$x_1 + x_2 \leq 2$$
$$5x_1 + 2x_2 \leq 10$$
$$3x_1 + 8x_2 \leq 12, \quad x_1, x_2 \geq 0 \qquad (6)$$

(c) Write the dual of the following problem :

Maximize $z = 2x + 3y + z$

Subject to the constraints :

$$4x + 3y + z = 6 ;$$

$$x + 2y + 5z = 4, \quad x, y, z \geq 0.$$ (6)

6. (a) Explain the concept of Dominance in the solution of rectangular games.

Use dominance method to solve the following game :

Player B

$$\text{Player A} \begin{bmatrix} 6 & 8 & 8 \\ 4 & 12 & 2 \end{bmatrix}$$ (3+3=6)

(b) Solve the following game Graphically :

Player B

$$\text{Player A} \begin{bmatrix} 1 & 2 \\ 5 & 4 \\ -7 & 9 \\ -4 & -3 \\ 2 & 1 \end{bmatrix}$$ (6)

(c) Solve the following (3×3) game by L.P. method.

Player B

$$\text{Player A} \begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 3 \\ -1 & 2 & -1 \end{bmatrix}$$ (6)

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