This question paper contains 16+4 printed pages]

Your Roll No................

7564

B.A. (Programme)/III D-I

MATHEMATICS—Paper III

(Selected Topics in Mathematics)

(NC—Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks* : 100

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions selecting *two* parts from each question. Unit I and Unit II are compulsory and contain *four* questions. In Unit III choose any of the options and attempt *two* questions from the same. Marks are indicated.

Unit I (Real Analysis)

1. (*a*) For any real numbers x, y : show that 8½

(*i*) $|x + y| \leq |x| + |y|$

(*ii*) $||x| - |y|| \leq |x - y|$

P.T.O.

(*b*) Define a closed set. Which of the following sets are closed ? Give arguments in support of your answer :

(*i*) the set $\mathbf{Z}$ of integers

(*ii*) $\{1, 2, 3\}$. 8½

(*c*) Define continuity of a function at a point. Show that a function f defined on $[0, 1]$ by

$$f(x) = \begin{cases} x & \text{when } x \text{ is rational} \\ 2x & \text{when } x \text{ is irrational} \end{cases}$$

is continuous only at $x = 0$. 8½

(*d*) Define uniform continuity of a function defined on an interval I. Show that every uniformly continuous function defined on an interval is continuous. Is the converse true ? Justify your answer. 8½

2. (*a*) If (a_n), (b_n) and (c_n) be three sequences such that :

(*i*) $a_n \leq b_n \leq c_n \quad \forall n$

(*ii*) $\lim_{n \to \infty} a_n = \lim_{n \to \infty} c_n = l$

then prove that (b_n) is also convergent and

$\lim_{n \to \infty} b_n = l$. 9

(*b*) If $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2a_n}$, $n > 1$, then show that (a_n) converges to 2. 9

(*c*) Define any *three* of the following and give examples of each :

(*i*) a bounded sequence

(*ii*) a convergent sequence

(*iii*) a divergent sequence

(*iv*) an oscillatory sequence. 9

(*d*) Test for convergence any *two* of the following series : 9

(*i*) $\frac{x^2}{2\sqrt{1}} + \frac{x^3}{3\sqrt{2}} + \ldots \quad x > 0$

(*ii*) $\sum \frac{n}{1 + 2^{-n}}$

(*iii*) $\sum \frac{n^{n^2}}{(n+1)^{n^2}}$.

3. (*a*) Show that :

$$\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi.$$ 8½

(*b*) Find the Fourier series of the function

$$f(x) = \begin{cases} 1 & \text{for } -\pi < x \le 0 \\ -2 & \text{for } 0 < x \le \pi \end{cases}.$$ 8½

(*c*) Find the radius of convergence of the following series : 8½

(*i*) $\sum \frac{(n!)^2 x^{2n}}{(2n)!}$

(*ii*) $\sum \frac{(n+1)}{(n+2)(n+3)} x^n$

(*d*) Show that the sequence (f_n) of functions, where

$$f_n(x) = \frac{n}{x+n}$$

is uniformly convergent in $[0, k]$ whatever k may be but not uniformly convergent in $[0, \infty)$. 8½

Unit II

(Computer Programming)

4. (*a*) (*i*) Write a program to implement logical operators. 3½

(*ii*) The following is a segment of a program : 3½

```
x = 1;
y = 1;
if (n > 0)
x = x + 1
y = y - 1
printf("%d %d", x, y);
```

What will be values of x and y if n assumes a value of (*a*) 1 (*b*) 0.

(*b*) (*i*) Write a short note on break statement. Explain its use with an example. 3½ (*ii*) Write a program that will read the value of x and evaluate the following function : 3½

$$y = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -1 & \text{for } x < 0 \end{cases}$$

(*c*) (*i*) Evaluate the following expressions float $a = 2.5$; $b = 2.5$;

(1) $a + 2.5/b + 4.5$

(2) $(a + 2.5)/b + 4.5$. 3½

(*ii*) Give the general form of the while loop. Explain how it works, with an example. 3½

Unit III (1)

(Numerical Analysis)

(Use of scientific calculator is allowed)

5. (*a*) Find the interval in which the smallest positive root of the equation

$$x^3 - x - 4 = 0 \text{ lies}$$

Determine the root correct to two decimal places, using the bisection method. 8½

(*b*) Solve the system of equation by Gauss Elimination method :

$$5x_1 - x_2 = 9$$

$$-x_1 + 5x_2 - x_3 = 4$$

$$-x_2 + 5x_3 = -6.$$ 8½

(*c*) Find the root of the equation

$$\sin x - \cosh x + 1 = 0,$$

correct to 4 decimal places, using Regula-falsi method. The root lies between 1 and 2. 8½

6. (*a*) Evaluate

$$I = \int_2^3 \frac{dx}{1+x}$$

using Gauss-Legendre integration method with $n = 4$. 8½

(*b*) Evaluate the integral $\int_0^2 \frac{e^{2x}}{1+x^2} dx$, using Simpson's 1/3 rule with spacing $h = 0.25$. 8½

(*c*) Find the unique polynomial of degree 2 or less, such that $f(0) = 1$, $f(1) = 3$, $f(3) = 55$, using the Newton divided difference interpolation. 8½

Unit III (2)

(Discrete Mathematics)

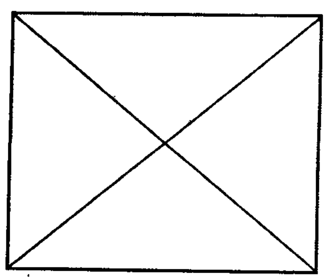
5. (*a*) Define the following giving examples : 8½

(*i*) The indegree and outdegree of a vertex in the directed graph.

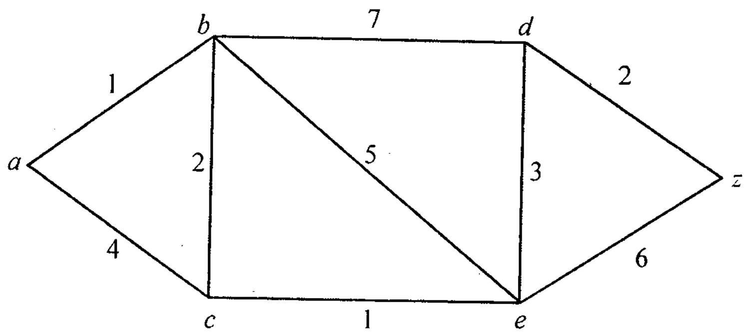
(*ii*) A multigraph

(*iii*) An Eulerian circuit.

(*b*) Is the following graph planar ? If no, draw the planar graph : 8½



(*c*) Find the shortest path from the vertex 'a' to vertex 'z' in the following weighted graph : 8½



6. (*a*) Show that the following statement is a tautology

$$(A \to B) \to [A \to (A \wedge B)].$$ 8½

(*b*) Find the combinatorial circuit corresponding to Boolean expression

$$(x_1 \wedge (\overline{x}_2 \vee x_3)) \vee x_2 .$$ 8½

(*c*) Write the following Boolean expression : 8½

$E(x_1, x_2, x_3)$

$= (x_1 \wedge x_2 \wedge \overline{x}_3) \vee (x_1 \wedge \overline{x}_2 \wedge x_4) \wedge (x_2 \wedge \overline{x}_3 \wedge \overline{x}_4)$

over the two valued Boolean algebra in conjunctive normal form.

Unit III (3)

(Statistics)

5. (*a*) Show that the mean deviation about the median is the minimum or least. 8½

(*b*) For two variables x and y, the two regression lines are 8½

$$x + 2y - 5 = 0$$

$$2x + 3y - 8 = 0.$$

If $\text{var}(x) = 12$, find $\overline{x}, \overline{y}, \sigma_y$ and r.

(*c*) Show that the line of best fit to the following data is given by 8½

$$y = 8 - 0.5x.$$

x	y
6	5
7	5
7	4
8	5
8	4
8	3
9	4
9	3
10	3

6. (*a*) The probability of n independent events are p_1, p_2, p_3, ..., p_n. Find an expression for the probability that at least one of the event will happen. Use the result to find the chance of obtaining at least one 6 in a throw of four dice. 8½

(*b*) Find the probability that at most 5 defective fuses will be found in a box of 200 fuses if experience shows that 2% of each fuse are defective. ($e^{-4} = 0.0183$). 8½

(*c*) Obtain the moment generating function about the origin of general normal distribution. 8½

Unit III (4)

(Mechanics)

5. (*a*) Three forces each equal to P act along the sides of a triangle ABC in order; prove that the resultant is

$$P\left(1 - 8 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}\right)^{1/2}$$ 8½

(*b*) A ladder of length $2a$, and weight W lb, with its centre of gravity $\frac{3}{8}$ of the way up it, stands on a smooth horizontal plane, resting against a smooth vertical wall and the middle point is tied to a point in the wall by horizontal rope of length l, find the tension of the rope and the reactions of the wall and the horizontal plane. 8½

(c) Six equal rods AB, BC, CD, DE, EF and FA are each of weight W and are freely jointed at their extremities so as to form a hexagon; the rod AB is fixed in a horizontal position, and the middle points of AB and DE are joined by a string; prove that the tension in the string is 3W. 8½

6. (a) A particle moving in a plane, describes the equiangular spiral $r = ae^{(\theta \cot \alpha)}$. If the radius vector to the particle has a constant angular velocity, show that the resultant

acceleration of the particle makes an angle 2α with the radius vector and is of magnitude $\frac{V^2}{r}$, where V is the speed. 8½

(*b*) A particle of mass m is placed on a horizontal board, which is made to execute vertical simple harmonic oscillations of period T and amplitude a. Show that the particle does not lose contact with the board at any time if $a < \left(\frac{gT^2}{4\pi^2}\right)$ 8½

(*c*) A shell is fired vertically upward, with speed V_0. The resistance is $mgcV^2$. Show that it attains its greatest height at time t given by

$$\tan(gt\sqrt{c}) = V_0\sqrt{c}.$$ 8½

Unit III (5)

(Game Theory)

5. (*a*) Solve the following Linear Programming Problem graphically :

Minimize $Z = 3x_1 + 2x_2$

Subject to the constraints :

$$x_1 - x_2 \leq 1$$

$$x_1 + x_2 \geq 3$$

$$x_1, x_2 \geq 0.$$

8½

(*b*) Use simplex method to solve the linear programming problem :

Maximize $Z = 3x + 4y$

Subject to the constraints :

$$3x + 2y \leq 18$$

$$x \leq 4$$

$$y \leq 6.$$

$$x, y \geq 0.$$

8½

(*c*) Develop the Dual of the following linear programming problem :

Minimize $Z = x_1 - 3x_2 - 3x_3$

Subject to the constraints :

$$3x_1 - x_2 + 2x_3 \leq 7$$

$$2x_1 - 4x_2 \geq 12$$

$$-4x_1 + 3x_2 + 8x_3 = 10$$

$$x_1, x_2 \geq 0$$

x_3 is unrestricted in sign. 8½

6. (*a*) Use Dominance Principle to solve the following game : 8½

Player B

Player A $\begin{bmatrix} 5 & -10 & 9 & 0 \\ 6 & 7 & 8 & 1 \\ 8 & 7 & 15 & 2 \\ 3 & 4 & -1 & 4 \end{bmatrix}$

(*b*) Obtain the optimal strategies for both persons and the value of the game for zero-sum two persons game whose pay-off matrix is as follows :

Player B

$$\text{Player A} \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix}$$

Use Graphical method. 8½

(*c*) Two companies A and B are competing for the same product. Their different strategies are given in the following pay-off matrix :

Company A

$$\text{Company B} \begin{bmatrix} 2 & -2 & 3 \\ -3 & 5 & -1 \end{bmatrix}$$

Use linear programming to determine the best strategies for both the players. 8½