This question paper contains 16+3 printed pages]

Your Roll No................

7565

B.A. Prog./III **D-I**

MATHEMATICS—Paper III

(Selected Topics in Mathematics)

(NC—Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks* : 100

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *All* questions selecting *two* parts from each question.

Unit I and Unit II are compulsory and contain four questions.

In Unit III choose any of the options and attempt *two* questions from the same. Marks are indicated.

Unit I

1. (*a*) Define limit point of a set $\mathbf{S} \subseteq \mathbf{R}$. Find the limit points of each of each of the following sets 8½

(*i*) $\mathbf{Z}$

(*ii*) $\{3^{-n} + 5^{-n} : n \in \mathbf{N}\}$

(*iii*) $\{1, 2, 3, 4\}$.

P.T.O.

(*b*) Define a neighbourhood of a point. Show that the intersection of two neighbourhoods of a point of **R** is again a neighbourhood of same point. What happens to the conclusion if arbitrary collection of neighbourhoods is taken ? Justify. 8½

(*c*) Define uniform continuity of a function. A function f is said to be contraction map if $\exists$ some $0 < \alpha < 1$ such that for any x, y in the domain : 8½

$$|f(x) - f(y)\ \mathrm{K}\alpha\,|x - y|$$

Show that a contraction map is uniformly continuous.

8½

(*d*) Let f be a function defined by : 8½

$$f(x) = \begin{cases} x \sin \dfrac{1}{x}, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ 2x \sin \dfrac{1}{x}, & \text{if } x < 0 \end{cases}$$

Discuss the continuity of f at $x = 0$.

2. (a) If $\lim_{n\to\infty} a_n = a$ and $a \neq 0$, show that there exists a real number K and a positive number m such that : 9

$$|a_n| > K \text{ whenever } n \geq m.$$

(b) Let (a_n) be a sequence where : 9

$$a_n = \left(1 + \frac{1}{n}\right)^n$$

Show that (a_n) is bounded and monotonically increasing.

(c) Show that an absolute convergent series is convergent. Give an example to show that a conditionally convergent series may not be absolutely convergent. 9

(d) Test for convergence for any *two* of the following series : 9

(i) $\frac{1}{5} + \frac{2!}{5^2} + \frac{3!}{5^3} + \ldots\ldots\ldots\ldots$

(ii) $\frac{1}{3} + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + \ldots\ldots\ldots$

(iii) $\Sigma \cos \frac{1}{n^2}$.

3. (*a*) Define a Riemann integrable function f defined on $[a, b]$. Give an example of each of the following kind : 8½

(*i*) Bounded but not Riemann integrable.

(*ii*) Riemann integrable but not continuous.

(*b*) Show that : 8½

$$\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx = \beta(m, n).$$

(*c*) Find the Fourier series of the following function : 8½

$$f(x) = \begin{cases} -1 & \text{for } -\pi \le x < 0 \\ 1 & \text{for } 0 \le x \le \pi \end{cases}$$

(*d*) Show that the sequence (f_n) where :

$$f_n(x) = x^n$$

is uniformly convergent in $[0, k]$, $k < 1$ and only pointwise convergent in $[0, 1]$. 8½

Unit II

(Computer Programming)

4. (*a*) (*i*) Calculate the final value of *b* in the following sequence of statements : 3½

```
float b;
int i;
b=2.56;
b=(b + 0.5) * 10;
i=b;
b=i;
b=b/10.0;
```

(*ii*) What is array ? Define the types of array briefly. 3½ (*b*) (*i*) Write C assignment statements to evaluate the following equations : 3½

(1) Area = $\pi r^2 + 2\pi r h$

(2) Torque = $\frac{2\, m_1 m_2}{m_1 + m_2} . g$.

(*ii*) Write a program to find whether a given point (x, y) lies on the x-axis, y-axis or at origin namely (0, 0). 3½

(*c*) (*i*) Write a short note on the break statement. Explain its use with an example. 3½

(*ii*) Write a program to find the sum of elements on the diagonal of a square matrix. 3½

Unit III(1)

(Numerical Analysis)

(Use of Scientific Calculator is allowed).

5. (*a*) Use the Newton-Raphson method to find a root of the equation : 8½

$$x^3 - 2x - 5 = 0.$$

(*b*) Solve the following system of equations by Gauss elimination method : 8½

$$4x + y + z = 4$$

$$x + 4y - 2z = 4$$

$$3x + 2y - 4z = 6.$$

(c) Show that the Gauss-Seidel method diverges for solving the system of equations : 8½

$$\begin{bmatrix} 1 & -5 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}.$$

6. (a) If

$$f(x) = \frac{-1}{x^3},$$

find the divided difference $f[x_0, x_1, x_2]$. 8½

(b) Given $p(-1) = -2$, $p(1) = 0$, $p(4) = 63$, $p(7) = 342$. Find the Lagrange interpolating polynomial which fits the given data. Estimate $p(5)$. 8½

(c) Evaluate : 8½

$$\int_0^{\pi/2} e^{\sin x} dx$$

using Simpson's $\frac{3}{8}$ rule with 6 sub-intervals.

Unit III(2)

(Discrete Mathematics)

5. (*a*) Define the following giving examples : 8½

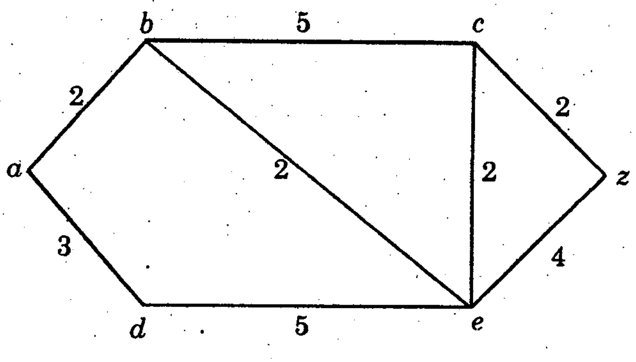
(*i*) A directed graph

(*ii*) A hamiltonian path

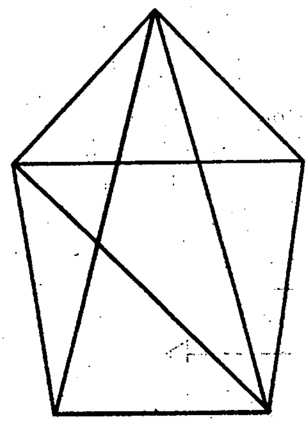
(*iii*) An Eulerian path

(*iv*) A subgraph of a graph.

(*b*) Find the length of a shortest path between *a* and *z* in the weighted graph : 8½



(*c*) Draw the planar graph of the following graph : 8½



6. (*a*) Define and illustrate the following : 8½

(*i*) Maxterm

(*ii*) Conjunctive normal form

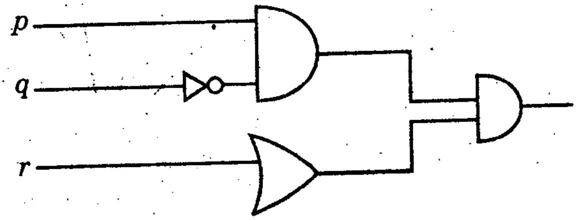
(*iii*) Disjunctive normal form

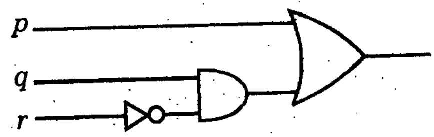
Write $(x_1 \wedge x_2) \vee (x_1 \wedge x_3) \vee (\bar{x}_2 \wedge x_3)$ in disjunctive normal form.

(*b*) Write the truth tables for $p \to q$ and $p \leftrightarrow q$ and show that : 8½

$$p \leftrightarrow q = (p \to q) \wedge (q \to p).$$

(*c*) Write Boolean expressions corresponding to the following circuits : 8½





Unit III(3)

(Statistics)

5. (*a*) The first four moments of a distribution about the value 4 are –1.5, 17, –30, 108. Find the moments about the mean β_1 and β_2. 8½

P.T.O.

(*b*) Fit a parabola of second degree to the following data : 8½

x	y
0	1
1	1.8
2	1.3
3	2.5
4	6.3

(*c*) Two independent variates x_1 and x_2 have means 5 and 10 and variances 4 and 9 respectively. Obtain the correlation coefficient between : 8½

$y_1 = 3x_1 + 4x_2$ and

$y_2 = 3x_1 - x_2$.

6. (*a*) Four persons are chosen at random from a group containing 3 male, 2 female and 4 children. Show that the chance that exactly two of them will be children is 10/21. 8½

(*b*) Find four moments of the Poisson distribution. 8½

Or

Find four moments of the Binomial Distribution.

(*c*) If X is normally distributed with mean 11 and standard deviation 1.5, find the number x_0 such that : 8½

(*i*) $P(X > x_0) = .3$ and

(*ii*) $P(X > x_0) = .09$.

Unit III(4)

(Mechanics)

5. (*a*) A ladder leans against a smooth wall, the lower end resting on a rough floor for which the coefficient of friction is $\frac{1}{4}$. Find the inclination of the ladder to the vertical if it is just on the point of slipping. 8½

(*b*) Two equal uniform rods AB and AC, each of length $2b$, are freely jointed at A and rest on a smooth vertical circle of radius a. Show that if 2θ is the angle between them, then : 8½

$$b \sin^3 \theta = a \cos \theta.$$

(*c*) Find the mass center of a wire bent into the form of an isosceles right-angled triangle. 8½

6. (*a*) A particle moves in a straight line with uniform acceleration and its distances from the origin O on the line (not necessarily the position at $t = \theta$) at time t_1, t_2, t_3 are d_1, d_2, d_3 respectively. Prove that if t_1, t_2, t_3 form an A.P. where common difference is d and d_1, d_2, d_3 are in G.P., then the acceleration is : 8½

$$\frac{\left(\sqrt{d_1} - \sqrt{d_3}\right)^2}{d^2}.$$

(*b*) A particle just clears a wall of height b at a distance a and strikes the ground at a distance c from the point of projection. Prove that the angle of projection is : 8½

$$\tan^{-1} \frac{bc}{a(c-a)}.$$

(c) A particle is performing a S.H.M. of period T about a centre O, and it passes through the position P(OP = b), with velocity V in the direction OP. Prove that the time which elapses before its return to P, is : 8½

$$\frac{T}{\pi}\tan^{-1}\left(\frac{VT}{2\pi ab}\right).$$

Unit III(5)

(Game Theory)

5. (a) Using Graphical method, find the maximum value of : 8½

$$Z = 7x + 10y$$

Subject to the constraints :

$$x + y \geq 30$$

$$y \leq 12$$

$$x \geq 6$$

$$x \geq y, \qquad x, y \geq 0.$$

(*b*) Solve the following Linear Programming Problem by Simplex method : 8½

Maximize :

$$Z = 3x + 2y$$

Subject to the constraints :

$$5x + 3y \leq 1600$$

$$3x + 3y \leq 1400$$

$$2x + 4y \leq 1200,$$

$$x, y \geq 0.$$

(*c*) Write the dual of the following problem : 8½

Minimize :

$$Z = x_1 + x_2 + x_3$$

Subject to the constraints :

$$x_1 - 3x_2 + 4x_3 = 5$$

$$x_1 - 2x_2 \leq 3$$

$$2x_2 - x_3 \geq 4,$$

$$x_1, x_2 \geq 0$$

x_3 is unrestricted in sign.

6. (*a*) Use dominance method to solve the following game : 8½

	I	II	III	IV
I	3	2	4	0
II	3	4	2	4
III	4	2	4	0
IV	0	4	0	8

(*b*) The following matrix represents the pay-off of T_1 in a rectangular game between two persons P_1 and P_2 : 8½

$$\begin{array}{cc} & \mathbf{P_2} \\ \mathbf{P_1} & \begin{bmatrix} 8 & 15 & -4 & -2 \\ 19 & 15 & 17 & 16 \\ 0 & 20 & 15 & 5 \end{bmatrix} \end{array}$$

By the notion of dominance, reduce the game to 2 × 4 game and solve it graphically.

(*c*) Solve the following game by linear programming : 8½

$$\begin{array}{cc} & \textbf{Player B} \\ \textbf{Player A} & \begin{bmatrix} 2 & 4 & 4 \\ 5 & 1 & 5 \\ 6 & 6 & 0 \end{bmatrix} \end{array}.$$