This question paper contains 4+1 printed pages]

Your Roll No.....

6744

B.A./B.Sc.(Hons)/III **D**

MATHEMATICS—Paper XIV

(Mechanics II)

(Admissions of 2008 and before)

Time : 2 Hours *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *All* questions, selecting *two* parts of each question.

Symbols have their usual meanings.

1. (*a*) For a particle moving along a plane curve, derive the radial and transverse components of velocity and acceleration at any time *t*. 5

(*b*) A bead of mass *m* slides on a smooth wire in the form of a parabola with axis vertical and vertex downwards. If the bead starts from rest at the end of the latus rectum (of length $4p$), find the speed with which it passes

P.T.O.

through the vertex. Find also the reactions of the wire on the bead at this point. 5

(*c*) A point moves so that it always has a velocity compounded of V_1 parallel to a fixed line and V_2 in the transverse direction perpendicular to OP, where O is a fixed point. Prove that its orbit is a conic having O as focus and discuss the cases $V_1 > V_2$, $V_1 < V_2$, $V_1 = V_2$. 5

2. (*a*) A particle of unit mass is projected with velocity V and inclination α in a medium whose resistance is k times velocity. Show that if k is small the path of the particle is approximately

$$Y = x \tan \alpha - gx^2/2V^2 \cos^2\alpha - kgx^3/3V^3\cos^3\alpha.$$ 5

(*b*) A shell is fired vertically upward with speed v_0. The resistance in $mgcv^2$ show that it attains its greatest height at time t given by $\tan (g\, t\sqrt{c}) = v_0\sqrt{c}$.

Deduce that no matter how large v_0 may be t cannot exceed $\frac{1}{2}\pi g^{-1}c^{-1/2}$. 5

(*c*) A particle of mass m oscillates on a line with natural period $2\pi/n$. If an applied periodic force $f \cos pt$ now acts in the line so that the particle is instantaneously at rest at zero time at a distance d from the centre of oscillation, prove that the displacement of the particle at a subsequent time t is

$$d \cos nt + f(\cos pt - \cos nt)/(n^2 - p^2)m. \quad 5$$

3. (*a*) Derive the first order differential equation of the central orbit. 4½

(*b*) If $P = \mu(u^2 - au^3)$ where $a > 0$ and a particle is projected from an apse at a distance a from the centre of force with a velocity $(\mu c/a^2)^{1/2}$, where $a > c$, prove that the other apsidal distance of the orbit is $a(a + c)/(a - c)$ and find the apsidal angle. 4½

(*c*) A particle of mass m moves in a central orbit under the force $P = mkr^{-2}e^{-r2}$, where k is a constant. Prove that a circular orbit of radius r is stable if and only if $r^2 < \frac{1}{2}$. 4½

4. (*a*) Derive an expression for kinetic energy of a rigid body with a fixed point rotating with constant angular velocity $\vec{\omega}$. 4½

(*b*) A square of side a has particle of masses m, $2m$, $3m$, $4m$ at its vertices. Show the principal axis that the principal moments of inertia at the centre of the square are $2ma^2$, $3\ ma^2$, $5\ ma^2$ and find the directions of the principal axes. 4½

(*c*) A rod of length $2a$ and mass m turns about one end O describing a cone with semi-vertical angle α. It completes a revolution in time T. Find the magnitude and direction of the angular momentum of the rod about O. 4½