This question paper contains 4 printed pages]

Your Roll No. ....

6747

B.A./B.Sc. (Hons.)/III D

MATHEMATICS—Paper XVII and XVIII (*i*)

(Number Theory)

Time : 2 *Hours* *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each Section.

All Sections are compulsory.

Section I

1. (*a*) A customer bought a dozen pieces of fruit, apples and oranges, for Rs. 132. If an apple cost 3 rupees more than an orange and more apples than oranges were purchased, how many pieces of each kind were bought ? 4½

P.T.O.

(*b*) Find the solution of the system of linear congruence equations : 4½

$x \equiv 5 \pmod{6}$, $x \equiv 4 \pmod{11}$, $x \equiv 3 \pmod{17}$.

(*c*) Prove that the quadratic congruence :

$$x^2 + 1 \equiv 0 \pmod{p}$$

where p is an odd prime has a solution if and only if $p \equiv 1 \pmod{4}$. 4½

Section II

2. (*a*) If the $n > 1$ has the prime factorization :

$$n = p_1^{k_1} \; p_2^{k_2} \; \ldots\ldots \; p_r^{k_r}.$$

then prove the following :

(*i*) $\sum_{d|n} \mu(d)\tau(d) = (-1)^r$ 2

(*ii*) $\sum_{d|n} \mu(d)\sigma(d) = (-1)^r \; p_1 p_2 \ldots\ldots p_r$ 3

(*b*) Prove that for $n > 2$, $\phi(n)$ is an even integer, and hence deduce Euclid theorem. 5

(*c*) Use the linear cipher $c \equiv 5p + 11 \pmod{26}$ to encrypt the message : 5

I WORK TO SUCCEED.

Section III

3. (*a*) Prove that if

$$\gcd(m, n) = 1$$

where $m > 2$ and $n > 2$, then the integer mn has no primitive roots. 5

(*b*) Let p be an odd prime and $\gcd(a, p) = 1$, then prove that a is quadratic residue of p if and only if : 5

$$a^{(p-1)/2} \equiv 1 \pmod{p}$$

(*c*) Find two primitive roots of 10. 5

Section IV

4. (*a*) (*i*) Show that the Fermat number F_5 is divisible by 641. 2½

(*ii*) If n is a perfect number, prove that : 2

$$\sum_{d|n} \frac{1}{d} = 2.$$

(*b*) Prove that an odd prime p is expressible as a sum of two squares if and only if 4½

$$p \equiv 1 \pmod 4.$$

(*c*) (*i*) Show that each of the integers 2^n,

where :

$$n = 1, 2, 3, \ldots\ldots,$$

is a sum of two squares. 2

(*ii*) Show that every odd perfect number (if one exists) is the sum of two squares. 2½